STOCHASTIC DIFFERENTIAL EQUATIONS (BMETE95MM08)
SRPING 2017
PRELIMINARY SCHEDULE

Week 01, 06 Feb – 12 Feb, and
Week 02, 13 Feb – 19 Feb, and
Week 03, 20 Feb – 26 Feb:

Brownian motion – phenomenology: Axioms, scaling of simple symmetric random walk, marginal distributions of BM, all about Gaussian random variables, path integral formulation.

Brownian motion – construction: Sketch of Wiener's construction, Levy's construction, relation to heat equation.

Brownian motion – further properties: Reflection, scaling, time-inversion. Sample path properties: no monotonicity, no differentiability at given point. Almost sure nowhere differentiability. Quadratic variation. Reflection principle.

Problems related: problem set 1
Assigned HW: **1.1, 1.2, 1.5, 1.13** **Due: 16 Feb**
1.6, 1.8, 1.10, 1.12 **Due: 02 Mar**

Week 04, 27 Feb – 05 Mar:

Filtrations and stopping times: Review.

Markov processes: Review.

Martingales: Martingales, submartingales, supermartingales. Optional stopping theorem. Maximal inequality.

Stochastic integration: Introduction, motivation, examples of SDE. Why not Riemann?

Problems related: problem set 2
Assigned HW: **2.4, 2.5, 2.6, 2.7** **Due: 09 Mar**

Week 05, 06 Mar – 12 Mar, and
Week 06, 13 Mar – 19 Mar:

Ito integral: Progressive measurability, Ito isometry, construction of Ito integral. Basic properties. Indefinite Ito integral as stochastic process. Multidimensional Ito integral. Ito processes. Quadratic variation.

Ito's formula: First cook-book, then rigorous proof. Multidimensional. Martingale representation theorem.

Problems related: problem set 3
Assigned HW: **3.1, 3.2, 3.3, 3.4** **Due: 16 Mar**
3.5, 3.7, 3.10, 3.12 **Due: 30 Mar**

Week 07, 20 Mar – 26 Mar:
Stochastic differential equations: Examples revisited, geometric Brownian motion, Ornstein-Uhlenbeck process, Bessel processes.
Strong solution: Existence and uniqueness under Lipschitz condition.

Problems related: problem set 4
Assigned HW: **4.2, 4.3, 4.4, 4.5** **Due: 06 Apr**

Week 08, 27 Mar – 02 Apr, and
Week 09, 03 Apr – 09 Apr:
Diffusions: Introduction, examples. Dynkin's formula. The infinitesimal generator. Applications of Dynkin's formula: recurrence/transience of BM in R^d.
Diffusions continued: Applications of Dynkin's formula to other prosesses. Kolmogorov's backward equation. Diffusions and parabolic pde-s. Kolmogorov's forward equation.
Diffusions and elliptic and parabolic PDEs. Solving Laplace-, Poisson-, Helmholtz-, etc equations with Brownian motions.
Feynman-Kac formula and some applications. Applications to **boundary value problems**.

Problems related: problem set 5
Assigned HW: **5.1, 5.3, 5.4, 5.6** **Due: 20 Apr**

Week 10, 10 Apr – 16 Apr, and
Week 11, 17 Apr – 23 Apr:
Diffusions continued – the semigroup approach: Estimates on displacement and on stability wrt initial point. The semigroup of conditional expectations. Feller property, strong continuity. Elements of theory of strongly continuous contraction semigroups on Banach spaces.
The importance of **boundary conditions**: BM on the line, BM on the half line {reflecting / absorbing / sticky} boundary conditions.

Problems related: problem set 6
Assigned HW: **6.1, 6.5, 6.6, 6.7** **Due: 4 May**

Week 12, 24 Apr – 30 Apr, and
Week 13, 01 May – 07 May:
Girsanov's formula.
Random change of time.
Weak solutions and **the martingale problem**.

Problems related: problem set 7
Assigned HW: **7.2, 7.3, 7.4, 7.8** **Due: 11 May**

Week 12, 08 May – 14 May:
General review …..