References for Hilversum lecture notes

Here follows the complete list of references (papers, surveys, books) mentioned during and relevant for the content of the lectures. The list follow the alphabetical order of authors, not the chronological or logical order of appearance of ideas.

References

[1] R. Arratia: The motion of a tagged particle in the simple symmetric exclusion system on $\mathbb{Z}$. *Ann. Probab.*, **24**: 362–373 (1983)

In this paper it is proved that the displacement of a tagged particle in 1-dimensional nearest neighbour symmetric simple exclusion process in equilibrium (steady-state) is *subdiffusive*. More precisely: the displacement scaled with fourth root (rather than square root) of time converges in distribution to a nondegenerate Gaussian.

[2] A. De Masi, P.A. Ferrari, S. Goldstein, D. Wick: An invariance principle for reversible Markov processes. Applications to random motions in random environments. *J. Stat. Phys.* **55**: 787-855 (1989)

The paper contains many nice applications of the original (reversible) Kipnis-Varadhan theorem. In particular the time-reversal trick for proving relevant uncorrelatedness of some random variables is exploited.

[3] R. Durrett: *Probability: Theory and Examples.* (Fourth Edition) Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 2010

Basic source for all background material in probability theory.

[4] M.I. Gordin, B.A. Lifshits: Central limit theorem for stationary Markov processes. *Dokladi Akademii Nauk SSSR* **239**: 766–767 (1978) [in Russian]

In this papere it is proved that if the square integrable function f is in the range of the infinitesimal generator of the semigroup acting on $\mathscr{L}^2(\Omega, \pi)$ then the central limit theorem folds for its integral along the trajectory of the Markov process. The theorem is technically absolutely starightforward, not more is needed than in the finite state space case. Nevertheless, the importance of the paper is due to the noveltyof approach: the martingale approximation appears here first.

[5] I. Horváth, B. Tóth, B. Vető: Diffusive limits for "true" (or myopic) self-avoiding random walks and self-repellent Brownian polymers in $d \geq 3$. *Probab. Theory Rel. Fields* **153**:

691-726 (2012)

`CLT is proved for the displacement of the so-called self-repelling Brownian polymer in` $\mathbb{R}^d$, $d \geq 3$, `and in a particular steady-state (for the occupation time profile, as seen from the moving particle). The proof uses the Graded Sector Condition. Also: the same type of result is proved for the lattice version: the so-called myopic self-avoiding walk on` $\mathbb{Z}^d$, $d \geq 3$. `The proof shows similarities with that in [10]The paper also contains a slighht improvement on the GSC.`

[6] I. Horváth, B. Tóth, B. Vető: Relaxed sector condition. *Bull. Inst. Math. Acad. Sin. (N.S.)* **7**: 463-476 (2012)

`The Relaxed Sector Condition (RSC) appears in this paper. It is also proved that thhe Graded Sector Condition implies RSC, and thus the condition for efficient martingale approximation is indeed relaxed.`

[7] C. Kipnis, S.R.S. Varadhan: Central limit theorem for additive functionals of reversible Markov processes with applications to simple exclusion, *Commun. Math. Phys.* **106**: 1-19 (1986)

`This is the celebrated paper where the first original reversible version of the Kipnis-Varadhhan theorem appears. As main application, diffusive limit (CLT and invariance principle) is proved for tagged particle diffusion in symmetric simple exclusion, in dimension 2 or more and in dimension 1 if the support of the jump distribution has at least four elements.`

[8] T. Komorowski, C. Landim, S. Olla: *Fluctuations in Markov Processes – Time Symmetry and Martingale Approximation. Grundlehren der mathematischen Wissenschaften*, Vol. 345, Springer, Berlin-Heidelberg-New York, 2012

`This is a recent monograph on Kipnis-Varadhan theory and its applications (and much more).`

[9] T. Komorowski, S. Olla: A note on the central limit theorem for two-fold stochastic random walks in a random environment. *Bull. Pol. Acad. Sci. Math.* **51**: 217-232 (2003)

`In this paper the random walk in divergence-free random drift field (or: bistochastic RWRE) is considered. However, as noted later in [8] the proof of CLT in the most general case is unfortunately incomplete.`

[10] T. Komorowski, S. Olla: On the sector condition and homogenization of diffusions with a Gaussian drift. *Journal of Functional Analysis* **197**: 179–211 (2003)

`CLT and homogenization is proved for diffusion in divergenc-free random, ergodic` *`Gaussian`* `drift field. The Graded Sector Condition is used.`

[11] S. M. Kozlov: The method of averaging and walks in inhomogeneous environments. *Uspekhi Mat. Nauk* **40**: 61-120 (1985) English version: *Russian Math. Surveys* **40**: 73-145 (1985)

`This is a survey on RWRE and homogenisation, up to date at its time. It contains many brilliant ideas and also some incomplete proofs.`

[12] G. Kozma, B. Toth: CLT for random walks in divergence-free random drift field: H_{-1} suffices. *preprint available soon from the arxiv*

CLT is proved for bistochastic RWRE under minimal integrability condition on the stream tensor.

[13] C. Landim, J. Quastel, M. Salmhofer, H-T. Yau: Superdiffusivity of asymmetric exclusion process in dimensions one and two. *Commun. Math. Phys.* **244**: 455–481 (2004)
By use of the variational formula for the asymptotic variance (see lecture 2) a method is provided for deriving lower bounds for the asymptotic variance of random displacements. As an application, supperdiffusive lower bound is proved for the displacement of the second class particle in asymmetric simple exclusion process.

[14] T. L. Liggett: *Interacting Particle Systems.* Grundlehren der Mathematischen Wissenschaften. vol **276**, Springer-Verlag, New York, 1985

The main source for rigorous mathematical theory of interacting particle systems on $\mathbb{Z}^d$.

[15] T. L. Liggett: *Continuous time Markov processes. An introduction.* Graduate Studies in Mathematics, vol. 113. American Mathematical Society, Providence, RI, 2010.

A best source for Markov processes, Feller property, semigroups, analytic approaches.

[16] M. Maxwell, M. Woodroofe: Central limit theorems for additive functionals of Markov chains *Annals of Probability* **28**: 713-724 (2000)

Yet another abstract theorem on CLT for additive functionals of Markov chains and processes. In [26] it is shown that the main result of this paper is direct consequnce of the non-reversible version of Kipnis-Varadhan theorem, cf [25]

[17] B. Morris, Y. Peres: Evolving sets, mixing and heat kernel bounds. *Probab. Theory Relat. Fields* **133**: 245-266 (2005)

This paper contains a combinatorial-flavour proof of a uniform-in-space upper bound for the heat-kernel of bistochastic random walks on $\mathbb{Z}^d$. The bound also follows in a neat and more robust way from Nash's inequality.

[18] K. Oelschläger: Homogenization of a diffusion process in a divergence-free random field, *Ann. Probab.* **16**: 1084-1126 (1988)

In this paper CLT is proved for diffusion in divergence-free ergodic random drift field in $\mathbb{R}^d$. This is the continuous-space (diffusion) counterpart of the result presented in lecture 5. The proof is based on strong technical details: cutoffs, approximations, heavy technicalities. It extends Osada's result [20] to the case of square integrable, but not necessarily bounded stream tensor. The proof in [12], based on Relaxed Sector Condition of [6] can also be extended to this case, providing a more transparent, less heavy-handed proof.

[19] S. Olla: Central limit theorems for tagged particles and for Diffusions in random environment. In: F. Comets, É. Pardoux (eds): Milieux aléatoires. *Panor. Synth.* **12**, Societé Mathématique de France, Paris, 2001

`This is a survey paper. It contains the skeleton of the theory and some applications. No fully exposed details, however.`

[20] H. Osada: Homogenization of diffusion processes with random stationary coefficients. In: Probability Theory and Mathematical Statistics (Tbilisi, 1982). *Lecture Notes in Mathematics* **1021**: 507-517, Springer, Berlin (1983)

In this paper the CLT is proved for diffusion in divergence-free random ergodic drift-filed in $\mathbb{R}^d$, with the assumption that the drift field is the curl of a *bounded* stream tensor.

[21] G.C. Papanicolaou, S.R.S. Varadhan: Boundary value problems with rapidly oscillating random coefficients. In: Fritz, J., Szász, D., Lebowitz, J.L. (eds.): Random Fields (Esztergom, 1979). *Colloq. Math. Soc. JĂĄnos Bolyai* **27**: 835-873, North-Holland, Amsterdam (1981)

The problem of homodenisation or CLT for diffusion in divergence-free random ergodic drift field originates here.

[22] M. Reed, B. Simon: *Methods of Modern Mathematical Physics Vol 1, 2.* Academic Press New York, 1975

The main recommended textbook for basics of functional analysis, operator theory, spectraL and resolvent calculus (including unbounded operators), self-adjointness, semigroups and their generators, etc.

[23] S. Sethuraman: A martingale CLT. (Lecture note, available form the S.S.'s homepage.)

A clear presentation of the martingale CLT, used in the proof of the main K-V theorem.

[24] S. Sethuraman, S.R.S. Varadhan, H-T. Yau: Diffusive limit of a tagged particle in asymmetric simple exclusion processes. *Comm. Pure Appl. Math.* **53**: 972-1006 (2000)

The Graded Sector Condition is formulated here and it is applied to tagged particle diffusion in asymmetric simple exclusion process with arbitrary (non-zero) mean, in $\mathbb{Z}^d$, $d \geq 3$.

[25] B. Tóth: Persistent random walk in random environment. *Probab. Theory Relat. Fields* **71**: 615-625 (1986)

The non-reversible version of Kipnis-Varadhan theorem appears here. In the proof spectral calculus is replaced by resolvent calculus. The theorem is applied to so-called persistent random walk in random environment, which is a kind of randomized lattice Lorentz gas.

[26] B. Tóth: Comment on a theorem of M. Maxwell and M. Woodroofe. *Electr. Commun. Probab.* **18**: paper no. 13, 1-4 (2013)

`It is proved that the main result of [16] is direct consequence of the non-reversible version of Kipnis-Varadhan theorem, cf [25].`

[27] P. Tarrès, B. Tóth, B. Valkó: Diffusivity bounds for 1d Brownian polymers. *Ann. Probab.* **40**: 695-713 (2012)

`Superdiffusive lower bounds are proved for self-repelling Brownian polymers with various self-interaction functions. The method initiated in [13] exploited and extended.`

[28] B. Tóth, B. Valkó: Superdiffusive bounds on self-repellent Brownian polymers and diffusion in the curl of the Gaussian free field in d=2. *Journ. Stat. Phys.* **147**: 113-131 (2012)

Superdiffusive lower bound (with $\log\log t$ correction) is proved for the displacement of self-repelling Brownian polymer and diffusion in the curl-field of (mollified) Gaussian free field, in $\mathbb{R}^2$. The method initiated in [13] exploited and extended.

[29] S.R.S. Varadhan: Self-diffusion of a tagged particle in equilibrium of asymmetric mean zero random walks with simple exclusion. *Ann. Inst. H. Poincaré Probab. Statist.* **31**: 273-285 (1996)

`The Strong Sector Condition is formulated in this paper and the corresponding theorem applied to tagged particle diffusion in zero-mean asymmetric simple exclusion process.`