Komputeralgebrai algoritmusok

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Ezek a programok csak szemléltetésre szolgálnak.

1. Történet

2. Algebrai alapok

```
> restart;
```

E 2.1. Példa.

```
> irem(1,1); irem(1,-1);
```

$$0$$

$$0 \quad (2.1.1)$$

```
> irem(18,6); irem(30,6);
```

$$0$$

$$0 \quad (2.1.2)$$

```
> irem(18,-6); irem(30,-6);
```

$$0$$

$$0 \quad (2.1.3)$$

```
> irem(6,-6); irem(-6,6);
```

$$0$$

$$0 \quad (2.1.4)$$

E 2.2. Példa.

```
> igcd(18,30);
```

$$6 \quad (2.2.1)$$

E 2.3. Példa.

```
> sign(-6); abs(6); sign(6); abs(6); sign(0); abs(0);
```

$$-1$$

$$6$$

```
1
6
1
0                                                      (2.3.1)
```

```
> igcd(-18,30); -18*30/igcd(-18,30); abs(-18*30)/igcd(-18,30);
  ilcm(-18,30);
```

```
6
-90
90
90                                                     (2.3.2)
```

E 2.4. Példa.

```
> abs(-6); abs(6);
```

```
6
6                                                      (2.4.1)
```

E 2.5. Példa.

```
> gcd(18,30);  2*18+(-1)*30; (-3)*18+2*30; 7*18+(-4)*30;
```

```
6
6
6
6                                                      (2.5.1)
```

A 2.1. Algoritmus.

```
> Euclid:=proc(a,b,x) local c,d,r,ua,ub,uc;
    if nargs<3 then
      c:=abs(a); d:=abs(b);
    else
      ua:=lcoeff(collect(a,x),x); if ua<>0 then c:=a/ua fi;
      ub:=lcoeff(collect(b,x),x); if ub<>0 then d:=b/ub fi;
    fi;
    while d<>0 do
      if nargs<3 then r:=irem(c,d) else r:=rem(c,d,x) fi;
      c:=d; d:=r;
    od;
    if nargs<3 then
      abs(c);
```

```
    else
      uc:=lcoeff(collect(c,x),x); if uc<>0 then c/uc else c fi;
    fi;
  end;
```

$Euclid := \mathbf{proc}(a, b, x)$ (2.6.1)

```
    local c, d, r, ua, ub, uc;
    if nargs < 3 then
        c := abs(a);
        d := abs(b)
    else
        ua := lcoeff(collect(a, x), x);
        if ua <> 0 then
            c := a / ua
        end if;
        ub := lcoeff(collect(b, x), x);
        if ub <> 0 then
            d := b / ub
        end if
    end if;
    while d <> 0 do
        if nargs < 3 then
            r := irem(c, d)
        else
            r := rem(c, d, x)
        end if;
        c := d;
        d := r
    end do;
    if nargs < 3 then
        abs(c)
    else
        uc := lcoeff(collect(c,
       x), x);
        if uc <> 0 then
            c / uc
        else
            c
        end if
    end if
```

```
end proc
```

E 2.7. Példa.

```
> debug(Euclid);
```

$$Euclid \tag{2.7.1}$$

```
> Euclid(18,30);
{--> enter Euclid, args = 18, 30
```

$$c := 18$$
$$d := 30$$
$$r := 18$$
$$c := 30$$
$$d := 18$$
$$r := 12$$
$$c := 18$$
$$d := 12$$
$$r := 6$$
$$c := 12$$
$$d := 6$$
$$r := 0$$
$$c := 6$$
$$d := 0$$
$$6$$

```
<-- exit Euclid (now at top level) = 6}
```

$$6 \tag{2.7.2}$$

A 2.2. Algoritmus.

```
> EEA:=proc(a,b,s,t,x) local c,c1,c2,d,d1,d2,q,r,r1,r2,ua,ub,
  uc;
    if nargs<5 then
      c:=abs(a); d:=abs(b);
    else
      ua:=lcoeff(collect(a,x),x); if ua<>0 then c:=a/ua fi;
      ub:=lcoeff(collect(b,x),x); if ub<>0 then d:=b/ub fi;
    fi;
    c1:=1; d1:=0; c2:=0; d2:=1;
    while d<>0 do
      if nargs<5 then q:=iquo(c,d) else q:=quo(c,d,x) fi;
      r:=expand(c-q*d);
```

```
      r1:=expand(c1-q*d1);
      r2:=expand(c2-q*d2);
      c:=d; c1:=d1; c2:=d2;
      d:=r; d1:=r1; d2:=r2;
    od;
    if nargs<5 then
      s:=c1/sign(a)/sign(c); t:=c2/sign(b)/sign(c); abs(c);
    else
      uc:=lcoeff(collect(c,x),x);
      if uc<>0 then
        if ua<>0 then s:=c1/uc/ua else s:=c1/uc fi;
        if ub<>0 then t:=c2/uc/ub else s:=c2/uc fi;
        c/uc;
      else
        if ua<>0 then s:=c1/ua else s:=c1 fi;
        if ub<>0 then t:=c2/ub else s:=c2 fi;
        c;
      fi;
    fi;
  end;
```

$$EEA := \textbf{proc}(a, b, s, t, x) \tag{2.8.1}$$

```
EEA := proc(a, b, s, t, x)
    local c, c1, c2, d, d1, d2, q, r, r1, r2, ua, ub, uc;
    if nargs < 5 then
        c := abs(a);
        d := abs(b)
    else
        ua := lcoeff(collect(a,
       x), x);
        if ua <> 0 then
            c := a / ua
        end if;
        ub := lcoeff(collect(b, x),
       x);
        if ub <> 0 then
            d := b / ub
        end if
    end if;
    c1 := 1;
    d1 := 0;
    c2 := 0;
    d2 := 1;
    while d <> 0 do
```

```
        if nargs < 5 then
            q := iquo(c, d)
        else
            q := quo(c, d, x)
        end if;
        r := expand(c - q*d);
        r1 := expand(c1 - q*d1);
        r2 := expand(c2 - q*d2);
        c := d;
        c1 := d1;
        c2 := d2;
        d := r;
        d1 := r1;
        d2 := r2
    end do;
    if nargs < 5 then
        s := c1 / (sign(a)*sign(c));
        t := c2 / (sign(b)*sign(c));
        abs(c)
    else
        uc := lcoeff(collect(c, x),
       x);
        if uc <> 0 then
            if ua <> 0 then
                s := c1 / (uc*ua)
            else
                s := c1 / uc
            end if;
            if ub <> 0 then
                t := c2 / (uc*ub)
            else
                s := c2 / uc
            end if;
            c / uc
        else
            if ua <> 0 then
                s := c1 / ua
            else
```

```
            s := c1
        end if;
        if ub <> 0 then
            t := c2 / ub
        else
            s := c2
        end if;
        c
    end if
  end if
end proc
```

E 2.8. Példa.

```
> debug(EEA);
```

$$EEA \quad (2.9.1)$$

```
> EEA(18,30,'s','t');
{--> enter EEA, args = 18, 30, s, t
```

$$c := 18$$
$$d := 30$$
$$c1 := 1$$
$$d1 := 0$$
$$c2 := 0$$
$$d2 := 1$$
$$q := 0$$
$$r := 18$$
$$r1 := 1$$
$$r2 := 0$$
$$c := 30$$
$$c1 := 0$$
$$c2 := 1$$
$$d := 18$$
$$d1 := 1$$
$$d2 := 0$$
$$q := 1$$
$$r := 12$$
$$r1 := -1$$
$$r2 := 1$$

$c := 18$

$c1 := 1$

$c2 := 0$

$d := 12$

$d1 := -1$

$d2 := 1$

$q := 1$

$r := 6$

$r1 := 2$

$r2 := -1$

$c := 12$

$c1 := -1$

$c2 := 1$

$d := 6$

$d1 := 2$

$d2 := -1$

$q := 2$

$r := 0$

$r1 := -5$

$r2 := 3$

$c := 6$

$c1 := 2$

$c2 := -1$

$d := 0$

$d1 := -5$

$d2 := 3$

$s := 2$

$t := -1$

6

```
<-- exit EEA (now at top level) = 6}
```

$$6 \tag{2.9.2}$$

```
> s; t;
```

$$2$$

$$-1 \tag{2.9.3}$$

E 2.9. Példa.

E 2.10. Példa.

E 2.11. Példa.

```
> a:=3*x^3+x^2+x+5; b:=5*x^2-3*x+1;
```

$$a := 3\,x^3 + x^2 + x + 5$$

$$b := 5\,x^2 - 3\,x + 1 \qquad (2.12.1)$$

```
> q1:=3/5*x; r1:=expand(a-q1*b);
```

$$q1 := \frac{3}{5}\,x$$

$$r1 := \frac{14}{5}\,x^2 + \frac{2}{5}\,x + 5 \qquad (2.12.2)$$

```
> q2:=14/25; r2:=expand(r1-q2*b);
```

$$q2 := \frac{14}{25}$$

$$r2 := \frac{52}{25}\,x + \frac{111}{25} \qquad (2.12.3)$$

```
> q:=q1+q2; r:=r2; a=expand(q*b+r);
```

$$q := \frac{3}{5}\,x + \frac{14}{25}$$

$$r := \frac{52}{25}\,x + \frac{111}{25}$$

$$3\,x^3 + x^2 + x + 5 = 3\,x^3 + x^2 + x + 5 \qquad (2.12.4)$$

E 2.12. Példa.

E 2.13. Példa.

E 2.14. Példa.

```
> a:=48*x^3-84*x^2+42*x-36; b:=-4*x^3-10*x^2+44*x-30;
```

$$a := 48\,x^3 - 84\,x^2 + 42\,x - 36$$

$$b := -4\,x^3 - 10\,x^2 + 44\,x - 30 \qquad (2.15.1)$$

```
> Euclid(a,b,x);
{--> enter Euclid, args = 48*x^3-84*x^2+42*x-36, -4*x^3
-10*x^2+44*x-30, x
```

$$ua := 48$$

$$c := x^3 - \frac{7}{4}\,x^2 + \frac{7}{8}\,x - \frac{3}{4}$$

$$ub := -4$$

$$d := x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2}$$

$$r := -\frac{33}{4} - \frac{17}{4}\,x^2 + \frac{95}{8}\,x$$

$$c := x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2}$$

$$d := -\frac{33}{4} - \frac{17}{4}\,x^2 + \frac{95}{8}\,x$$

$$r := -\frac{1605}{578} + \frac{535}{289}\,x$$

$$c := -\frac{33}{4} - \frac{17}{4}\,x^2 + \frac{95}{8}\,x$$

$$d := -\frac{1605}{578} + \frac{535}{289}\,x$$

$$r := 0$$

$$c := -\frac{1605}{578} + \frac{535}{289}\,x$$

$$d := 0$$

$$uc := \frac{535}{289}$$

$$-\frac{3}{2} + x$$

```
<-- exit Euclid (now at top level) = -3/2+x}
```

$$-\frac{3}{2} + x \qquad (2.15.2)$$

E 2.15. Példa.

```
> EEA(a,b,'s','t',x);
{--> enter EEA, args = 48*x^3-84*x^2+42*x-36, -4*x^3-10*
x^2+44*x-30, s, t, x
```

$$ua := 48$$

$$c := x^3 - \frac{7}{4}\,x^2 + \frac{7}{8}\,x - \frac{3}{4}$$

$$ub := -4$$

$$d := x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2}$$

$$c1 := 1$$
$$d1 := 0$$
$$c2 := 0$$
$$d2 := 1$$
$$q := 1$$
$$r := -\frac{33}{4} - \frac{17}{4} x^2 + \frac{95}{8} x$$
$$r1 := 1$$
$$r2 := -1$$
$$c := x^3 + \frac{5}{2} x^2 - 11 x + \frac{15}{2}$$
$$c1 := 0$$
$$c2 := 1$$
$$d := -\frac{33}{4} - \frac{17}{4} x^2 + \frac{95}{8} x$$
$$d1 := 1$$
$$d2 := -1$$
$$q := -\frac{4}{17} x - \frac{360}{289}$$
$$r := -\frac{1605}{578} + \frac{535}{289} x$$
$$r1 := \frac{360}{289} + \frac{4}{17} x$$
$$r2 := -\frac{71}{289} - \frac{4}{17} x$$
$$c := -\frac{33}{4} - \frac{17}{4} x^2 + \frac{95}{8} x$$
$$c1 := 1$$
$$c2 := -1$$
$$d := -\frac{1605}{578} + \frac{535}{289} x$$
$$d1 := \frac{360}{289} + \frac{4}{17} x$$
$$d2 := -\frac{71}{289} - \frac{4}{17} x$$
$$q := -\frac{4913}{2140} x + \frac{3179}{1070}$$
$$r := 0$$

$$r1 := -\frac{289}{107} + \frac{1156}{535}x + \frac{289}{535}x^2$$

$$r2 := -\frac{289}{1070} + \frac{289}{2140}x - \frac{289}{535}x^2$$

$$c := -\frac{1605}{578} + \frac{535}{289}x$$

$$c1 := \frac{360}{289} + \frac{4}{17}x$$

$$c2 := -\frac{71}{289} - \frac{4}{17}x$$

$$d := 0$$

$$d1 := -\frac{289}{107} + \frac{1156}{535}x + \frac{289}{535}x^2$$

$$d2 := -\frac{289}{1070} + \frac{289}{2140}x - \frac{289}{535}x^2$$

$$uc := \frac{535}{289}$$

$$s := \frac{3}{214} + \frac{17}{6420}x$$

$$t := \frac{71}{2140} + \frac{17}{535}x$$

$$-\frac{3}{2} + x$$

```
<-- exit EEA (now at top level) = -3/2+x}
```

$$-\frac{3}{2} + x \qquad (2.16.1)$$

```
> s; t; expand(s*a+t*b);
```

$$\frac{3}{214} + \frac{17}{6420}x$$

$$\frac{71}{2140} + \frac{17}{535}x$$

$$-\frac{3}{2} + x \qquad (2.16.2)$$

E 2.16. Példa.

```
> p:=5*x^3*y^2-x^2*y^4-3*x^2*y^2+7*x*y^2+2*x*y-2*x+4*y^4+5;
```

$$p := 5x^3y^2 - x^2y^4 - 3x^2y^2 + 7xy^2 + 2xy - 2x + 4y^4 + 5 \qquad (2.17.1)$$

```
> sort(p,[y,x],plex);
```

(2.17.2)

$$-y^4 x^2 + 4 y^4 + 5 y^2 x^3 - 3 y^2 x^2 + 7 y^2 x + 2 y x - 2 x + 5 \quad (2.17.2)$$

```
> sort(p,[x,y],plex);
```

$$5 x^3 y^2 - x^2 y^4 - 3 x^2 y^2 + 7 x y^2 + 2 x y - 2 x + 4 y^4 + 5 \quad (2.17.3)$$

E 2.17. Példa.

```
> collect(p,[x,y]);
```

$$5 x^3 y^2 + \left(-y^4 - 3 y^2\right) x^2 + \left(2 y - 2 + 7 y^2\right) x + 4 y^4 + 5 \quad (2.18.1)$$

E 2.18. Példa.

```
> p:=collect(p,[x,y],`distributed`); lcoeff(p,[x,y],'t'); t;
```

$$p := 5 x^3 y^2 - x^2 y^4 - 3 x^2 y^2 + 7 x y^2 + 2 x y - 2 x + 4 y^4 + 5$$

$$5$$

$$x^3 y^2 \quad (2.19.1)$$

```
> convert(t,list); map(x->op(2,x),%);
```

$$\left[x^3, y^2\right]$$

$$\left[3, 2\right] \quad (2.19.2)$$

```
> degree(p,{x,y});
```

$$6 \quad (2.19.3)$$

```
> degree(p,x);
```

$$3 \quad (2.19.4)$$

```
> degree(p,y);
```

$$4 \quad (2.19.5)$$

E 2.19. Példa.

```
> a=expand((1)*(2)*(3)*(2*x-3)*(4*x^2-x+2));
  b=expand((-1)*(2)*(2*x-3)*(x-1)*(x+5));
```

$$48 x^3 - 84 x^2 + 42 x - 36 = 48 x^3 - 84 x^2 + 42 x - 36$$

$$-4 x^3 - 10 x^2 + 44 x - 30 = -4 x^3 - 10 x^2 + 44 x - 30 \quad (2.20.1)$$

```
> expand((2)*(2*x-3));
```

$$-6 + 4 x \quad (2.20.2)$$

E 2.20. Példa.

```
> a=expand((48)*(x-3/2)*(x^2-1/4*x+1/2));
  b=expand((-4)*(x-3/2)*(x-1)*(x+5));
```

$$48\,x^3-84\,x^2+42\,x-36=48\,x^3-84\,x^2+42\,x-36$$
$$-4\,x^3-10\,x^2+44\,x-30=-4\,x^3-10\,x^2+44\,x-30 \quad (2.21.1)$$

```
> x-3/2;
```

$$-\frac{3}{2}+x \quad (2.21.2)$$

E 2.21. Példa.

```
> u:=proc(p,L,typ) local pp,uu;
    pp:=expand(p); pp:=collect(pp,L,`distributed`);
    uu:=lcoeff(pp,L);
    if uu=0 then return 1 fi;
    if typ='integer' then return sign(uu) fi;
    uu;
  end;
```

$u := \mathbf{proc}(p, L, typ)$ (2.22.1)

```
    local pp, uu;
    pp := expand(p);
    pp := collect(pp, L,
    distributed);
    uu := lcoeff(pp, L);
    if uu = 0 then
        return 1
    end if;
    if typ = 'integer' then
        return sign(uu)
    end if;
    uu
end proc
```

```
> cont:=proc(p,L,typ) local pp,uu,cL;
    if nops(L)=1 and typ<>'integer' then return 1 fi;
    uu:=u(p,L,typ);
    pp:=simplify(p/uu);
    pp:=collect(pp,L[1]);
    cL:=coeffs(pp,L[1]);
    if nops(L)=1 then return igcd(cL) fi;
    GCD([cL],L[2..nops(L)],typ);
  end;
```

$cont := \mathbf{proc}(p, L, typ)$ (2.22.2)

local *pp*, *uu*, *cL*;
if *nops*(*L*) = 1 **and** *typ* <> '*integer*' **then**
return 1
end if;
uu := *u*(*p*,
L, *typ*);
pp := *simplify*(*p* / *uu*);
pp := *collect*(*pp*, *L*[1]);
cL := *coeffs*(*pp*, *L*[1]);
if *nops*(*L*) = 1 **then**
return *igcd*(*cL*)
end if;
GCD([*cL*], *L*[2 ..*nops*(*L*)], *typ*)
end proc

```
> pp:=proc(p,L,typ) local uu,pp,c;
    uu:=u(p,L,typ);
    pp:=simplify(p/uu);
    c:=cont(pp,L,typ);
    if c=0 then 0 else simplify(pp/c) fi;
  end;
```

pp := **proc**(*p*, *L*, *typ*) (2.22.3)
local *uu*, *pp*, *c*;
uu := *u*(*p*, *L*, *typ*);
pp := *simplify*(*p* / *uu*);
c := *cont*(*pp*, *L*, *typ*);
if *c* = 0 **then**
0
else
simplify(*pp* / *c*)
end if
end proc

```
> a;
```

$$48\,x^3 - 84\,x^2 + 42\,x - 36 \quad (2.22.4)$$

```
> u(a,[x],'integer');
```

$$1 \quad (2.22.5)$$

```
> cont(a,[x],'integer');
```

$$6 \quad (2.22.6)$$

```
> pp(a,[x],'integer');
```

$$8\,x^3 - 14\,x^2 + 7\,x - 6 \quad (2.22.7)$$

```
> u(a,[x],'rational');
```

$$48 \tag{2.22.8}$$

```
> cont(a,[x],'rational');
```

$$1 \tag{2.22.9}$$

```
> pp(a,[x],'rational');
```

$$x^3 - \frac{7}{4}\,x^2 + \frac{7}{8}\,x - \frac{3}{4} \tag{2.22.10}$$

```
> b;
```

$$-4\,x^3 - 10\,x^2 + 44\,x - 30 \tag{2.22.11}$$

```
> u(b,[x],'integer');
```

$$-1 \tag{2.22.12}$$

```
> cont(b,[x],'integer');
```

$$2 \tag{2.22.13}$$

```
> pp(b,[x],'integer');
```

$$2\,x^3 + 5\,x^2 - 22\,x + 15 \tag{2.22.14}$$

```
> u(b,[x],'rational');
```

$$-4 \tag{2.22.15}$$

```
> cont(b,[x],'rational');
```

$$1 \tag{2.22.16}$$

```
> pp(b,[x],'rational');
```

$$x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2} \tag{2.22.17}$$

A 2.3. Algoritmus.

```
> pseudodiv:=proc(a,b,x,q,r) local l,beta,qq,aa,bb;
    aa:=collect(expand(a),x);
    bb:=collect(expand(b),x);
    l:=degree(aa,x)-degree(bb,x)+1;
    q:=0;
    if l<=0 then r:=aa; return fi;
    beta:=lcoeff(bb,x);
    aa:=collect(expand(aa*beta^l),x);
    while degree(aa,x)>=degree(bb,x) do
      l:=degree(aa,x)-degree(bb,x);
      qq:=lcoeff(aa,x)/beta;
      q:=q+qq;
      aa:=collect(expand(aa-qq*x^l*bb),x);
    od;
    r:=aa;
  end;
```

$$pseudodiv := \mathbf{proc}(a, b, x, q, r) \tag{2.23.1}$$

local *l*, beta, *qq*, *aa*, *bb*;
aa := *collect*(*expand*(*a*), *x*);
bb := *collect*(*expand*(*b*), *x*);
l := *degree*(*aa*, *x*) − *degree*(*bb*, *x*) + 1;
q := 0;
if *l* <= 0 **then**
r := *aa*;
return
end if;
beta := *lcoeff*(*bb*, *x*);
aa := *collect*(*expand*(*aa**beta^*l*), *x*);
while *degree*(*bb*,
x) <= *degree*(*aa*, *x*) **do**
l := *degree*(*aa*, *x*) − *degree*(*bb*, *x*);
qq := *lcoeff*(*aa*, *x*) / beta;
q := *q* + *qq*;
aa := *collect*(*expand*(*aa* − *qq** *x*^*l** *bb*), *x*)
end do;
r := *aa*
end proc

```
> PrimitiveEuclidean:=proc(a,b,L,typ) local c,d,r,q,gamma;
    c:=pp(a,L,typ); d:=pp(b,L,typ);
    while d<>0 do
      pseudodiv(c,d,L[1],'q','r');
      c:=d; d:=pp(r,L,typ);
    od;
    if nops(L)=1 then
      if typ='integer' then
        gamma:=igcd(cont(a,L,typ),cont(b,L,typ));
      else gamma:=1 fi;
    else
      gamma:=PrimitiveEuclidean(cont(a,L,typ),cont(b,L,typ),L
  [2..nops(L)],typ);
    fi; gamma*c;
  end;
```

PrimitiveEuclidean := **proc**(*a*, *b*, *L*, *typ*) (2.23.2)
local *c*, *d*, *r*, *q*, gamma;
c := *pp*(*a*,
L, *typ*);
d := *pp*(*b*, *L*, *typ*);
while *d* <> 0 **do**

```
            pseudodiv(c, d, L[1], 'q',
           'r');
            c := d;
            d := pp(r, L, typ)
      end do;
      if nops(L) = 1 then
           if typ = 'integer' then
                gamma := igcd(cont(a, L, typ), cont(b, L, typ))
           else
                gamma := 1
           end if
      else
           gamma := PrimitiveEuclidean(cont(a, L, typ), cont(b, L, typ),
          L[2 ..nops(L)], typ)
      end if;
      gamma * c
 end proc
```

```
> GCD:=proc(P,L,typ)
    if nops(P)=0 then return 0 fi;
    if nops(P)=1 then return expand(P[1]/u(P[1],L,typ)) fi;
    if nops(P)=2 then PrimitiveEuclidean(op(P),L,typ)
    else GCD([PrimitiveEuclidean(P[1],P[2],L,typ),op(P[3..nops
  (P)])],L,typ) fi;
  end;
```

```
GCD := proc(P, L, typ)                                                (2.23.3)
      if nops(P) = 0 then
           return 0
      end if;
      if nops(P) = 1 then
           return expand(P[1] / u(P[1], L, typ))
      end if;
      if nops(P) = 2 then
           PrimitiveEuclidean(op(P), L, typ)
      else
           GCD([PrimitiveEuclidean(P[1], P[2], L, typ), op(P[3 ..nops(P)])]
               ,L,
          typ)
      end if
 end proc
```

E 2.22. Példa.

```
> debug(PrimitiveEuclidean);
```

$$PrimitiveEuclidean \quad (2.24.1)$$

```
> PrimitiveEuclidean(a,b,[x],'integer');
{--> enter PrimitiveEuclidean, args = 48*x^3-84*x^2+42*x
-36, -4*x^3-10*x^2+44*x-30, [x], integer
```

$$c := 8\,x^3 - 14\,x^2 + 7\,x - 6$$

$$d := 2\,x^3 + 5\,x^2 - 22\,x + 15$$

$$-132 - 68\,x^2 + 190\,x$$

$$c := 2\,x^3 + 5\,x^2 - 22\,x + 15$$

$$d := 66 + 34\,x^2 - 95\,x$$

$$-6420 + 4280\,x$$

$$c := 66 + 34\,x^2 - 95\,x$$

$$d := -3 + 2\,x$$

$$0$$

$$c := -3 + 2\,x$$

$$d := 0$$

$$\gamma := 2$$

$$-6 + 4\,x$$

```
<-- exit PrimitiveEuclidean (now at top level) = -6+4*x}
```

$$-6 + 4\,x \quad (2.24.2)$$

E 2.23. Példa.

```
> aa:=-30*x^3*y+90*x^2*y^2+15*x^2-60*x*y+45*y^2;
  bb:=100*x^2*y-140*x^2-250*x*y^2+350*x*y-150*y^3+210*y^2;
```

$$aa := -30\,x^3\,y + 90\,x^2\,y^2 + 15\,x^2 - 60\,x\,y + 45\,y^2$$

$$bb := 100\,x^2\,y - 140\,x^2 - 250\,x\,y^2 + 350\,x\,y - 150\,y^3 + 210\,y^2 \quad (2.25.1)$$

```
> aa:=collect(aa,[x,y]);
```

$$aa := -30\,x^3\,y + \left(90\,y^2 + 15\right)x^2 - 60\,x\,y + 45\,y^2 \quad (2.25.2)$$

```
> bb:=collect(bb,[x,y]);
```

$$bb := \left(100\,y - 140\right)x^2 + \left(-250\,y^2 + 350\,y\right)x - 150\,y^3 + 210\,y^2 \quad (2.25.3)$$

```
> coeffs(aa,x);
```

$$45\,y^2,\ -60\,y,\ -30\,y,\ 90\,y^2 + 15 \quad (2.25.4)$$

```
> coeffs(bb,x);
```

$$-150\, y^3 + 210\, y^2,\ -250\, y^2 + 350\, y,\ 100\, y - 140 \qquad (2.25.5)$$

```
> debug(GCD):
> GCD([%%%],[y],'integer');
{--> enter GCD, args = [45*y^2, -60*y, -30*y, 90*y^2+15],
[y], integer
{--> enter PrimitiveEuclidean, args = 45*y^2, -60*y, [y],
integer
```

$$c := y^2$$

$$d := y$$

$$0$$

$$c := y$$

$$d := 0$$

$$\gamma := 15$$

$$15\, y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter GCD, args = [15*y, -30*y, 90*y^2+15], [y],
integer
{--> enter PrimitiveEuclidean, args = 15*y, -30*y, [y],
integer
```

$$c := y$$

$$d := y$$

$$0$$

$$c := y$$

$$d := 0$$

$$\gamma := 15$$

$$15\, y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter GCD, args = [15*y, 90*y^2+15], [y], integer
{--> enter PrimitiveEuclidean, args = 15*y, 90*y^2+15, [y]
, integer
```

$$c := y$$

$$d := 6\, y^2 + 1$$

$$c := 6\, y^2 + 1$$

$$d := y$$

$$1$$

$$c := y$$

$$d := 1$$

$$0$$

$$c := 1$$

$$d := 0$$

$$\gamma := 15$$
$$15$$
```
<-- exit PrimitiveEuclidean (now in GCD) = 15}
```
$$15$$
```
<-- exit GCD (now in GCD) = 15}
```
$$15$$
```
<-- exit GCD (now in GCD) = 15}
```
$$15$$
```
<-- exit GCD (now at top level) = 15}
```
$$15 \tag{2.25.6}$$

> GCD([%%%],[y],'integer');
```
{--> enter GCD, args = [-150*y^3+210*y^2, -250*y^2+350*y,
100*y-140], [y], integer
{--> enter PrimitiveEuclidean, args = -150*y^3+210*y^2,
-250*y^2+350*y, [y], integer
```
$$c := 5\,y^3 - 7\,y^2$$
$$d := 5\,y^2 - 7\,y$$
$$0$$
$$c := 5\,y^2 - 7\,y$$
$$d := 0$$
$$\gamma := 10$$
$$50\,y^2 - 70\,y$$
```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y^2-70*y}
{--> enter GCD, args = [50*y^2-70*y, 100*y-140], [y],
integer
{--> enter PrimitiveEuclidean, args = 50*y^2-70*y, 100*y
-140, [y], integer
```
$$c := 5\,y^2 - 7\,y$$
$$d := 5\,y - 7$$
$$0$$
$$c := 5\,y - 7$$
$$d := 0$$
$$\gamma := 10$$
$$50\,y - 70$$
```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y-70}
```
$$50\,y - 70$$
```
<-- exit GCD (now in GCD) = 50*y-70}
```
$$50\,y - 70$$
```
<-- exit GCD (now at top level) = 50*y-70}
```
$$50\,y - 70 \tag{2.25.7}$$

> undebug(GCD);

$$GCD \tag{2.25.8}$$

```
> pp(aa,[x,y],'integer');
{--> enter PrimitiveEuclidean, args = -45*y^2, 60*y, [y],
integer
```

$$c := y^2$$

$$d := y$$

$$0$$

$$c := y$$

$$d := 0$$

$$\gamma := 15$$

$$15\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter PrimitiveEuclidean, args = 15*y, 30*y, [y],
integer
```

$$c := y$$

$$d := y$$

$$0$$

$$c := y$$

$$d := 0$$

$$\gamma := 15$$

$$15\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter PrimitiveEuclidean, args = 15*y, -90*y^2-15,
[y], integer
```

$$c := y$$

$$d := 6\,y^2 + 1$$

$$c := 6\,y^2 + 1$$

$$d := y$$

$$1$$

$$c := y$$

$$d := 1$$

$$0$$

$$c := 1$$

$$d := 0$$

$$\gamma := 15$$

$$15$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15}
```

$$2\,x^3\,y - 6\,x^2\,y^2 - x^2 + 4\,x\,y - 3\,y^2 \tag{2.25.9}$$

```
> pp(bb,[x,y],'integer');
```

```
{--> enter PrimitiveEuclidean, args = -150*y^3+210*y^2,
-250*y^2+350*y, [y], integer
```

$$c := 5\,y^3 - 7\,y^2$$

$$d := 5\,y^2 - 7\,y$$

$$0$$

$$c := 5\,y^2 - 7\,y$$

$$d := 0$$

$$\gamma := 10$$

$$50\,y^2 - 70\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y^2-70*y}
{--> enter PrimitiveEuclidean, args = 50*y^2-70*y, 100*y
-140, [y], integer
```

$$c := 5\,y^2 - 7\,y$$

$$d := 5\,y - 7$$

$$0$$

$$c := 5\,y - 7$$

$$d := 0$$

$$\gamma := 10$$

$$50\,y - 70$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y-70}
```

$$-3\,y^2 - 5\,x\,y + 2\,x^2 \qquad (2.25.10)$$

```
> PrimitiveEuclidean(aa,bb,[x,y],'integer');
{--> enter PrimitiveEuclidean, args = -30*x^3*y+(90*
y^2+15)*x^2-60*x*y+45*y^2, (100*y-140)*x^2+(-250*y^2+350*
y)*x-150*y^3+210*y^2, [x, y], integer
{--> enter PrimitiveEuclidean, args = -45*y^2, 60*y, [y],
integer
```

$$c := y^2$$

$$d := y$$

$$0$$

$$c := y$$

$$d := 0$$

$$\gamma := 15$$

$$15\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter PrimitiveEuclidean, args = 15*y, 30*y, [y],
integer
```

$$c := y$$

$$d := y$$

$$0$$

$$c := y$$
$$d := 0$$
$$\gamma := 15$$
$$15\, y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter PrimitiveEuclidean, args = 15*y, -90*y^2-15,
[y], integer
```

$$c := y$$
$$d := 6\, y^2 + 1$$
$$c := 6\, y^2 + 1$$
$$d := y$$
$$1$$
$$c := y$$
$$d := 1$$
$$0$$
$$c := 1$$
$$d := 0$$
$$\gamma := 15$$
$$15$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15}
```

$$c := 2\, x^3\, y - 6\, x^2\, y^2 - x^2 + 4\, x\, y - 3\, y^2$$

```
{--> enter PrimitiveEuclidean, args = -150*y^3+210*y^2,
-250*y^2+350*y, [y], integer
```

$$c := 5\, y^3 - 7\, y^2$$
$$d := 5\, y^2 - 7\, y$$
$$0$$
$$c := 5\, y^2 - 7\, y$$
$$d := 0$$
$$\gamma := 10$$
$$50\, y^2 - 70\, y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y^2-70*y}
{--> enter PrimitiveEuclidean, args = 50*y^2-70*y, 100*y
-140, [y], integer
```

$$c := 5\, y^2 - 7\, y$$
$$d := 5\, y - 7$$
$$0$$
$$c := 5\, y - 7$$
$$d := 0$$
$$\gamma := 10$$

$$50\,y-70$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y-70}
```

$$d:=-3\,y^2-5\,x\,y+2\,x^2$$

$$\left(2\,y^3+6\,y\right)x-18\,y^2-6\,y^4$$

$$c:=-3\,y^2-5\,x\,y+2\,x^2$$

```
{--> enter PrimitiveEuclidean, args = -18*y^2-6*y^4, 2*
y^3+6*y, [y], integer
```

$$c:=3\,y^2+y^4$$

$$d:=y^3+3\,y$$

$$0$$

$$c:=y^3+3\,y$$

$$d:=0$$

$$\gamma:=2$$

$$2\,y^3+6\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 2*y^3+6*y}
```

$$d:=-3\,y+x$$

$$0$$

$$c:=-3\,y+x$$

$$d:=0$$

```
{--> enter PrimitiveEuclidean, args = -45*y^2, 60*y, [y],
integer
```

$$c:=y^2$$

$$d:=y$$

$$0$$

$$c:=y$$

$$d:=0$$

$$\gamma:=15$$

$$15\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
{--> enter PrimitiveEuclidean, args = 15*y, 30*y, [y],
integer
```

$$c:=y$$

$$d:=y$$

$$0$$

$$c:=y$$

$$d:=0$$

$$\gamma:=15$$

$$15\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15*y}
```

```
{--> enter PrimitiveEuclidean, args = 15*y, -90*y^2-15,
[y], integer
```

$$c := y$$
$$d := 6\,y^2 + 1$$
$$c := 6\,y^2 + 1$$
$$d := y$$
$$1$$
$$c := y$$
$$d := 1$$
$$0$$
$$c := 1$$
$$d := 0$$
$$\gamma := 15$$
$$15$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 15}
{--> enter PrimitiveEuclidean, args = -150*y^3+210*y^2,
-250*y^2+350*y, [y], integer
```

$$c := 5\,y^3 - 7\,y^2$$
$$d := 5\,y^2 - 7\,y$$
$$0$$
$$c := 5\,y^2 - 7\,y$$
$$d := 0$$
$$\gamma := 10$$
$$50\,y^2 - 70\,y$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y^2-70*y}
{--> enter PrimitiveEuclidean, args = 50*y^2-70*y, 100*y
-140, [y], integer
```

$$c := 5\,y^2 - 7\,y$$
$$d := 5\,y - 7$$
$$0$$
$$c := 5\,y - 7$$
$$d := 0$$
$$\gamma := 10$$
$$50\,y - 70$$

```
<-- exit PrimitiveEuclidean (now in GCD) = 50*y-70}
{--> enter PrimitiveEuclidean, args = 15, 50*y-70, [y],
integer
```

$$c := 1$$
$$d := 5\,y - 7$$

$$c := 5\,y - 7$$

$$d := 1$$

$$0$$

$$c := 1$$

$$d := 0$$

$$\gamma := 5$$

$$5$$

```
<-- exit PrimitiveEuclidean (now in PrimitiveEuclidean) =
5}
```

$$\gamma := 5$$

$$-15\,y + 5\,x$$

```
<-- exit PrimitiveEuclidean (now at top level) = -15*y+5*
x}
```

$$-15\,y + 5\,x \qquad (2.25.11)$$

E 2.24. Példa.

```
> PrimitiveEuclidean(a,b,[x],'rational');
{--> enter PrimitiveEuclidean, args = 48*x^3-84*x^2+42*x
-36, -4*x^3-10*x^2+44*x-30, [x], rational
```

$$c := x^3 - \frac{7}{4}\,x^2 + \frac{7}{8}\,x - \frac{3}{4}$$

$$d := x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2}$$

$$-\frac{33}{4} - \frac{17}{4}\,x^2 + \frac{95}{8}\,x$$

$$c := x^3 + \frac{5}{2}\,x^2 - 11\,x + \frac{15}{2}$$

$$d := \frac{33}{17} + x^2 - \frac{95}{34}\,x$$

$$-\frac{1605}{578} + \frac{535}{289}\,x$$

$$c := \frac{33}{17} + x^2 - \frac{95}{34}\,x$$

$$d := -\frac{3}{2} + x$$

$$0$$

$$c := -\frac{3}{2} + x$$

$$d := 0$$

$$\gamma := 1$$

$$-\frac{3}{2} + x$$

```
<-- exit PrimitiveEuclidean (now at top level) = -3/2+x}
```

$$-\frac{3}{2} + x \qquad (2.26.1)$$

E 2.25. Példa.

```
> -2/4; 2/(-4); 100/(-200); -600/1200;
```

$$-\frac{1}{2}$$

$$-\frac{1}{2}$$

$$-\frac{1}{2}$$

$$-\frac{1}{2} \qquad (2.27.1)$$

E 2.26. Példa.

```
> a:=17/100*x^2-3/112*x+1/2; b:=5/9*x^2+4/5;
```

$$a := \frac{17}{100} x^2 - \frac{3}{112} x + \frac{1}{2}$$

$$b := \frac{5}{9} x^2 + \frac{4}{5} \qquad (2.28.1)$$

```
> a/b;
```

$$\frac{\frac{17}{100} x^2 - \frac{3}{112} x + \frac{1}{2}}{\frac{5}{9} x^2 + \frac{4}{5}} \qquad (2.28.2)$$

```
> expand(a*25200)/expand(b*25200);
```

$$\frac{4284 x^2 - 675 x + 12600}{14000 x^2 + 20160} \qquad (2.28.3)$$

```
> expand(a*25200/14000)/expand(b*25200/14000);
```

$$\frac{\frac{153}{500} x^2 - \frac{27}{560} x + \frac{9}{10}}{x^2 + \frac{36}{25}} \qquad (2.28.4)$$

```
> normal(expand(a/b));
```
$$\frac{9}{560}\,\frac{476\,x^2-75\,x+1400}{25\,x^2+36} \quad (2.28.5)$$

```
> simplify(a/b);
```
$$\frac{9}{560}\,\frac{476\,x^2-75\,x+1400}{25\,x^2+36} \quad (2.28.6)$$

E 2.27. Példa.

```
> d:=series(1/(1-x),x);
```
$$d := 1+x+x^2+x^3+x^4+x^5+\mathrm{O}\left(x^6\right) \quad (2.29.1)$$

E 2.28. Példa.

```
> series(1/d,x);
```
$$1-x+\mathrm{O}\left(x^6\right) \quad (2.30.1)$$

E 2.29. Példa.

```
> with(powseries);
```
[*compose, evalpow, inverse, multconst, multiply, negative, powadd, powcos, powcreate, powdiff, powexp, powint, powlog, powpoly, powsin, powsolve, powsqrt, quotient, reversion, subtract, tpsform*] (2.31.1)

```
> c:='c'; powcreate(c(n)=1,c(0)=2,c(1)=0,c(2)=0); tpsform(c,x,
8);
```
$$c := c$$
$$2+x^3+x^4+x^5+x^6+x^7+\mathrm{O}\left(x^8\right) \quad (2.31.2)$$

```
> d:=powpoly(1-x,x); tpsform(d,x,8);
```
$d :=$ **proc**$(powparm)$ ... **end proc**
$$1-x \quad (2.31.3)$$

```
> e:=inverse(d); tpsform(e,x,8);
```
$e :=$ **proc**$(powparm)$ ... **end proc**
$$1+x+x^2+x^3+x^4+x^5+x^6+x^7+\mathrm{O}\left(x^8\right) \quad (2.31.4)$$

```
> a:=multiply(e,c); tpsform(a,x,8);
```
$a :=$ **proc**$(powparm)$ ... **end proc**
$$2+2\,x+2\,x^2+3\,x^3+4\,x^4+5\,x^5+6\,x^6+7\,x^7+\mathrm{O}\left(x^8\right) \quad (2.31.5)$$

```
> b:=multiply(a,e); tpsform(b,x,8);
```

$$b := \mathbf{proc}(powparm)\ \ldots\ \mathbf{end\ proc}$$

$$2 + 4x + 6x^2 + 9x^3 + 13x^4 + 18x^5 + 24x^6 + 31x^7 + O(x^8) \quad (2.31.6)$$

E 2.30. Példa.

```
> b:=powlog(d); tpsform(b,x,8); a:=negative(b); tpsform(a,x,8);
```

$$b := \mathbf{proc}(powparm)\ \ldots\ \mathbf{end\ proc}$$

$$-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \frac{1}{6}x^6 - \frac{1}{7}x^7 + O(x^8)$$

$$a := \mathbf{proc}(powparm)\ \ldots\ \mathbf{end\ proc}$$

$$x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5 + \frac{1}{6}x^6 + \frac{1}{7}x^7 + O(x^8) \quad (2.32.1)$$

```
> c:=multiply(a,e); tpsform(c,x,8);
```

$$c := \mathbf{proc}(powparm)\ \ldots\ \mathbf{end\ proc}$$

$$x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \frac{137}{60}x^5 + \frac{49}{20}x^6 + \frac{363}{140}x^7 + O(x^8) \quad (2.32.2)$$

E 2.31. Példa.

```
> c:='c'; powcreate(c(n)=1/2^(n-2),c(0)=0,c(1)=0); tpsform(c,x,
  8);
```

$$c := c$$

$$x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{1}{8}x^5 + \frac{1}{16}x^6 + \frac{1}{32}x^7 + O(x^8) \quad (2.33.1)$$

```
> a:=inverse(c);
Error, (in powseries:-inverse) inverse will have pole at zero
> series(1/(x^2*(1-x/2)),x);
```

$$x^{-2} + \frac{1}{2}x^{-1} + \frac{1}{4} + \frac{1}{8}x + \frac{1}{16}x^2 + \frac{1}{32}x^3 + \frac{1}{64}x^4 + \frac{1}{128}x^5 + O(x^6) \quad (2.33.2)$$

```
>
```

3. Normál formák, reprezentáció

4. Aritmetika

5. Kínai maradékolás

- **6. Newton–iteráció, Hensel–felemelés**
- **7. Legnagyobb közös osztó**
- **8. Faktorizálás**
- **9. Egyenletrendszerek**
- **10. Gröbner–bázisok**
- **11. Racionális törtfüggvények integrálása**
- **12. A Risch–algoritmus.**