Komputeralgebrai algoritmusok

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak.

▶ **1. Történet**

▶ **2. Algebrai alapok**

▶ **3. Normál formák, reprezentáció**

▶ **4. Aritmetika**

▶ **5. Kínai maradékolás**

▶ **6. Newton–iteráció, Hensel–felemelés**

▶ **7. Legnagyobb közös osztó**

▶ **8. Faktorizálás**

▶ **9. Egyenletrendszerek**

▶ **10. Gröbner–bázisok**

▶ **11. Racionális törtfüggvények integrálása**

▼ **12. A Risch–algoritmus.**

```
> restart;
> (x*(x+1)*((x^2*exp(2*x^2)-(log(x+1))^2)^2+2*x*exp(3*x^2)*(x-(2*
  x^3+2*x^2+x+1)*log(x+1))))/((x+1)*(log(x+1))^2-(x^3+x^2)*exp(2*
  x^2))^2;
```

$$\frac{1}{\left((x+1)\ln(x+1)^2 - \left(x^3+x^2\right)e^{2x^2}\right)^2}\left(x\,(x+1)\left(\left(x^2\,e^{2x^2}-\ln(x+1)^2\right)^2\right.\right. \tag{12.1}$$

$$\left.\left. + 2\,x\,e^{3x^2}\left(x - \left(2\,x^3+2\,x^2+x+1\right)\ln(x+1)\right)\right)\right)$$

```
> int(%,x);
```

$$x - \ln(x+1) + \frac{e^{x^2} x \ln(x+1)}{x^2 e^{2x^2} - \ln(x+1)^2} + \frac{1}{2}\ln\!\left(e^{x^2} x + \ln(x+1)\right) - \frac{1}{2}\ln\!\left(-e^{x^2} x\right) \quad (12.2)$$
$$+ \ln(x+1)\big)$$

```
> int(1/(1+exp(x)),x); simplify(%,symbolic);
```

$$-\ln\!\left(1 + e^x\right) + \ln\!\left(e^x\right)$$
$$-\ln\!\left(1 + e^x\right) + x \quad (12.3)$$

```
> int(x/(1+exp(x)),x);
```

$$\frac{1}{2}x^2 - x\ln\!\left(1 + e^x\right) - \mathrm{polylog}\!\left(2, -e^x\right) \quad (12.4)$$

```
> int(1/(1+x^2),x); convert(%,ln);
```

$$\arctan(x)$$
$$\frac{1}{2}\,\mathrm{I}\left(\ln(1 - \mathrm{I}x) - \ln(1 + \mathrm{I}x)\right) \quad (12.5)$$

```
> int(cos(x),x); convert(%,exp);
```

$$\sin(x)$$
$$-\frac{1}{2}\,\mathrm{I}\left(e^{\mathrm{I}x} - \frac{1}{e^{\mathrm{I}x}}\right) \quad (12.6)$$

```
> int(1/sqrt(1-x^2),x); convert(%,ln);
```

$$\arcsin(x)$$
$$-\mathrm{I}\ln\!\left(\sqrt{1 - x^2} + \mathrm{I}x\right) \quad (12.7)$$

```
> int(arccosh(x),x); convert(arccosh(x),ln); int(%,x);
```

$$x\,\mathrm{arccosh}(x) - \sqrt{x-1}\,\sqrt{x+1}$$
$$\ln\!\left(x + \sqrt{x-1}\,\sqrt{x+1}\right)$$
$$\ln\!\left(x + \sqrt{x-1}\,\sqrt{x+1}\right)x - \sqrt{x-1}\,\sqrt{x+1} \quad (12.8)$$

▼ E 12.1. Példa.

```
> f:=exp(x)+exp(2*x)+exp(x/2); int(f,x);
```

$$f := e^x + e^{2x} + e^{\frac{1}{2}x}$$
$$e^x + \frac{1}{2}e^{2x} + 2e^{\frac{1}{2}x} \quad (12.1.1)$$

▼ E 12.2. Példa.

```
> g:=sqrt(log(x^2+3*x+2)*(log(x+1)+log(x+2)));
```

$$(12.2.1)$$

$$g := \sqrt{\ln\left(x^2 + 3\,x + 2\right)\left(\ln\left(x+1\right) + \ln\left(x+2\right)\right)} \qquad (12.2.1)$$

▼ E 12.3. Példa.

```
> h:=exp(ln(x)/2);
```

$$h := \sqrt{x} \qquad (12.3.1)$$

▼ E 12.4. Példa.

```
> g:=log(sqrt(x^2+1)+x)+log(sqrt(x^2+1)-x); theta1:=sqrt(x^2+1)
  ;
  h:=(theta1-x)^k*(theta1+x)^k1; diff(h,x)/h; simplify(%);
```

$$g := \ln\left(\sqrt{x^2+1} + x\right) + \ln\left(\sqrt{x^2+1} - x\right)$$

$$\theta 1 := \sqrt{x^2+1}$$

$$h := \left(\sqrt{x^2+1} - x\right)^k \left(\sqrt{x^2+1} + x\right)^{k1}$$

$$\frac{1}{\left(\sqrt{x^2+1} - x\right)^k \left(\sqrt{x^2+1} + x\right)^{k1}}\left(\frac{\left(\sqrt{x^2+1} - x\right)^k k \left(\dfrac{x}{\sqrt{x^2+1}} - 1\right)\left(\sqrt{x^2+1} + x\right)^{k1}}{\sqrt{x^2+1} - x} + \frac{\left(\sqrt{x^2+1} - x\right)^k \left(\sqrt{x^2+1} + x\right)^{k1} k1 \left(\dfrac{x}{\sqrt{x^2+1}} + 1\right)}{\sqrt{x^2+1} + x}\right)$$

$$-\frac{k - k1}{\sqrt{x^2+1}} \qquad (12.4.1)$$

▼ E 12.5. Példa.

```
> exp(x^2+log(x)/2)/(2*(sqrt(x)-1))+exp(x^2+log(x)/2)/(2*(sqrt
  (x)+1));
  diff(%,x); simplify(%);
```

$$\frac{e^{x^2 + \frac{1}{2}\ln(x)}}{2\sqrt{x} - 2} + \frac{e^{x^2 + \frac{1}{2}\ln(x)}}{2\sqrt{x} + 2}$$

$$\frac{\left(2x+\dfrac{1}{2x}\right)\mathrm{e}^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}-2}-\frac{\mathrm{e}^{x^2+\frac{1}{2}\ln(x)}}{\left(2\sqrt{x}-2\right)^2\sqrt{x}}$$

$$+\frac{\left(2x+\dfrac{1}{2x}\right)\mathrm{e}^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}+2}-\frac{\mathrm{e}^{x^2+\frac{1}{2}\ln(x)}}{\left(2\sqrt{x}+2\right)^2\sqrt{x}}$$

$$\frac{\mathrm{e}^{x^2}\left(2x^3-2x^2-1\right)}{\left(\sqrt{x}-1\right)^2\left(\sqrt{x}+1\right)^2} \tag{12.5.1}$$

```
> int((2*x^3-2*x^2-1)/(x-1)^2*exp(x^2),x);
```

$$\frac{x\,\mathrm{e}^{x^2}}{x-1} \tag{12.5.2}$$

▼ E 12.6. Példa.

```
> int(1/ln(x),x);
```

$$-\mathrm{Ei}\left(1,-\ln(x)\right) \tag{12.6.1}$$

```
> resultant(1-z/x,theta,theta);
```

$$\frac{x-z}{x} \tag{12.6.2}$$

▼ E 12.7. Példa.

```
> int(1/x/ln(x),x);
```

$$\ln\left(\ln(x)\right) \tag{12.7.1}$$

```
> resultant(1/x-z/x,theta,theta);
```

$$-\frac{-1+z}{x} \tag{12.7.2}$$

```
> oR:=1-z;
```

$$oR:=1-z \tag{12.7.3}$$

```
> v1:=gcd(1/x-1/x,theta);
```

$$v1:=\theta \tag{12.7.4}$$

▼ E 12.8. Példa.

```
> (x*(x+1)*((x^2*exp(2*x^2)-(log(x+1))^2)^2+2*x*exp(3*x^2)*(x-
  (2*x^3+2*x^2+x+1)*log(x+1))))/((x+1)*(log(x+1))^2-(x^3+x^2)*
  exp(2*x^2))^2;
```

$$\frac{1}{\left(\left(x+1\right)\ln(x+1)^2-\left(x^3+x^2\right)\mathrm{e}^{2x^2}\right)^2}\left(x\left(x+1\right)\left(\left(x^2\,\mathrm{e}^{2x^2}-\ln(x+1)^2\right)^2\right.\right. \tag{12.8.1}$$

$$+ 2\,x\,\mathrm{e}^{3\,x^2}\left(x - \left(2\,x^3 + 2\,x^2 + x + 1\right)\ln(x+1)\right)\Big)\Big)$$

`> int(%,x);`

$$x - \ln(x+1) + \frac{\mathrm{e}^{x^2}\,x\,\ln(x+1)}{x^2\,\mathrm{e}^{2\,x^2} - \ln(x+1)^2} - \frac{1}{2}\,\ln\!\left(\ln(x+1) - x\,\mathrm{e}^{x^2}\right) \tag{12.8.2}$$
$$+ \frac{1}{2}\,\ln\!\left(x\,\mathrm{e}^{x^2} + \ln(x+1)\right)$$

`> subs(exp(x^2)=theta1,%%);`

$$\frac{1}{\left(\left(x+1\right)\ln(x+1)^2 - \left(x^3 + x^2\right)\mathrm{e}^{2\,x^2}\right)^2}\left(x\,(x+1)\left(\left(x^2\,\mathrm{e}^{2\,x^2} - \ln(x+1)^2\right)^2\right.\right. \tag{12.8.3}$$
$$\left.\left.+ 2\,x\,\mathrm{e}^{3\,x^2}\left(x - \left(2\,x^3 + 2\,x^2 + x + 1\right)\ln(x+1)\right)\right)\right)$$

`> subs(exp(2*x^2)=theta1^2,%);`

$$\frac{1}{\left(\left(x+1\right)\ln(x+1)^2 - \left(x^3 + x^2\right)\theta1^2\right)^2}\left(x\,(x+1)\left(\left(x^2\,\theta1^2 - \ln(x+1)^2\right)^2\right.\right. \tag{12.8.4}$$
$$\left.\left.+ 2\,x\,\mathrm{e}^{3\,x^2}\left(x - \left(2\,x^3 + 2\,x^2 + x + 1\right)\ln(x+1)\right)\right)\right)$$

`> subs(exp(3*x^2)=theta1^3,%);`

$$\frac{1}{\left(\left(x+1\right)\ln(x+1)^2 - \left(x^3 + x^2\right)\theta1^2\right)^2}\left(x\,(x+1)\left(\left(x^2\,\theta1^2 - \ln(x+1)^2\right)^2\right.\right. \tag{12.8.5}$$
$$\left.\left.+ 2\,x\,\theta1^3\left(x - \left(2\,x^3 + 2\,x^2 + x + 1\right)\ln(x+1)\right)\right)\right)$$

`> f:=subs(ln(x+1)=theta2,%);`

$$f := \frac{x\,(x+1)\left(\left(x^2\,\theta1^2 - \theta2^2\right)^2 + 2\,x\,\theta1^3\left(x - \left(2\,x^3 + 2\,x^2 + x + 1\right)\theta2\right)\right)}{\left(\left(x+1\right)\theta2^2 - \left(x^3 + x^2\right)\theta1^2\right)^2} \tag{12.8.6}$$

`> convert(f,parfrac,theta2,sqrfree);`

$$\frac{x}{x+1} - \frac{2\,x^2\,\theta1^3\left(2\,\theta2\,x^3 + 2\,\theta2\,x^2 + \theta2\,x + \theta2 - x\right)}{\left(-x^2\,\theta1^2 + \theta2^2\right)^2\,(x+1)} \tag{12.8.7}$$

`> a:=(2*x^2+1)*theta1*theta2-x/(x+1)*theta1;`

$$a := \left(2\,x^2 + 1\right)\theta1\,\theta2 - \frac{x\,\theta1}{x+1} \tag{12.8.8}$$

`> b:=theta2^2-x^2*theta1^2;`

$$b := -x^2\,\theta1^2 + \theta2^2 \tag{12.8.9}$$

`> oR:=4*z^2-1; c1:=1/2; c2:=-1/2;`

$$oR := 4\,z^2 - 1$$
$$c1 := \frac{1}{2}$$
$$c2 := -\frac{1}{2} \tag{12.8.10}$$

`> v1:=theta2+x*theta1; v2:=theta2-x*theta1;`

$$v1 := \theta2 + x\,\theta1$$
$$v2 := \theta2 - x\,\theta1 \tag{12.8.11}$$

▼ **E 12.9. Példa.**

>

▼ **E 12.10. Példa.**

>

▼ **E 12.11. Példa.**

>

▼ **E 12.12. Példa.**

>

▼ **E 12.13. Példa.**

>

▼ **E 12.14. Példa.**

>

▼ **E 12.15. Példa.**

>

▼ **E 12.16. Példa.**

>

▼ **E 12.17. Példa.**

>

▼ **E 12.18. Példa.**

>

▼ **E 12.19. Példa.**

>

▼ **E 12.20. Példa.**

>

▼ **E 12.21. Példa.**

>

▼ **E 12.22. Példa.**

>