Kalkulus II.

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak

1. Lineáris algebra

```
> restart;
```

1.1. Mátrixok és vektorok

1.1.1. Test.

```
> 1,2+2,3^2,3.45/3,Pi,3+4*I,x^2=-1,Alma,infinity,-infinity;
```

$$1,\ 4,\ 9,\ 1.150000000,\ \pi,\ 3+4\,I,\ x^2=-1,\ Alma,\ \infty,\ -\infty \tag{1.1.1.1}$$

```
> evalf(%);
```

$$1.,\ 4.,\ 9.,\ 1.150000000,\ 3.141592654,\ 3.+4.\,I,\ x^2=-1.,\ Alma,\ Float(\infty),\ -Float(\infty) \tag{1.1.1.2}$$

```
> %%[5],%%[6];
```

$$\pi,\ 3+4\,I \tag{1.1.1.3}$$

```
> ?Pi
> Digits:=50;
```

$$Digits := 50 \tag{1.1.1.4}$$

```
> evalf(Pi);
```

$$3.1415926535897932384626433832795028841971693993751 \tag{1.1.1.5}$$

```
> Digits:=10;
```

$$Digits := 10 \tag{1.1.1.6}$$

```
> A:=Alma; Alma:=1; A; Alma:='Alma'; A;
```

$$A := Alma$$

$$Alma := 1$$

$$1$$

$$Alma := Alma$$

$$Alma \tag{1.1.1.7}$$

```
> whattype(1); whattype(1/2); whattype(0.5); whattype(Alma);
  whattype(infinity);whattype(Pi);whattype(A);
  whattype(2); whattype(krikszkraksz); whattype
  ("krikszkraksz");
```

integer

fraction

float

symbol

extended_numeric

symbol

symbol

integer

symbol

string (1.1.1.8)

```
> type(1,integer);type(1,float);type(1+3*I,complexcons);
```

true

false

true (1.1.1.9)

```
> z:=3+4*I;Re(z);Im(z);abs(z);
```

$$z := 3 + 4\,I$$

3

4

5 (1.1.1.10)

```
> solve(x^2=-1);
```

$$I, -I \quad (1.1.1.11)$$

```
> solve(x^5+x+1=0);
```

$$-\frac{1}{2} + \frac{1}{2} I\sqrt{3}, -\frac{1}{2} - \frac{1}{2} I\sqrt{3}, \quad (1.1.1.12)$$

$$-\frac{1}{6}\left(100 + 12\sqrt{69}\right)^{1/3} - \frac{2}{3\left(100 + 12\sqrt{69}\right)^{1/3}} + \frac{1}{3},$$

$$\frac{1}{12}\left(100 + 12\sqrt{69}\right)^{1/3} + \frac{1}{3\left(100 + 12\sqrt{69}\right)^{1/3}} + \frac{1}{3}$$

$$+ \frac{1}{2} I\sqrt{3}\left(-\frac{1}{6}\left(100 + 12\sqrt{69}\right)^{1/3} + \frac{2}{3\left(100 + 12\sqrt{69}\right)^{1/3}}\right),$$

$$\frac{1}{12}\left(100+12\sqrt{69}\right)^{1/3}+\frac{1}{3\left(100+12\sqrt{69}\right)^{1/3}}$$
$$+\frac{1}{3}-\frac{1}{2}\,I\sqrt{3}\left(-\frac{1}{6}\left(100+12\sqrt{69}\right)^{1/3}\right.$$
$$\left.+\frac{2}{3\left(100+12\sqrt{69}\right)^{1/3}}\right)$$

```
> solve(x^5+x^2+1=0);
```

$$\mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=1\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=2\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=3\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=4\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=5\right) \quad (1.1.1.13)$$

```
> solve(x^7+x^6+x^5+x^4+x^3+2*x^2+x+1);
```

$$-\frac{1}{2}+\frac{1}{2}\,I\sqrt{3},\ -\frac{1}{2}-\frac{1}{2}\,I\sqrt{3},\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=1\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=2\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=3\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=4\right),\ \mathrm{RootOf}\left(_Z^5+_Z^2+1,\ index=5\right) \quad (1.1.1.14)$$

```
> evalf(%);
```

$$-0.5000000000+0.8660254040\,I,\ -0.5000000000-0.8660254040\,I,\ 0.7515192324+0.7846159210\,I,\ -0.1545896767+0.8280741332\,I,\ -1.193859111,\ -0.1545896767-0.8280741332\,I,\ 0.7515192324-0.7846159210\,I \quad (1.1.1.15)$$

1.1.2. Példa.

```
> `&+`:=(x,y)->x+y mod 2;
  0&+0; 0&+1; 1&+0; 1&+1;
```

$$\&+:=(x,y)\to x+y \bmod 2$$

$$0,\ 1,\ 1,\ 0 \quad (1.1.2.1)$$

1.1.3. Algebrai struktúrák.

1.1.4. Példák.

1.1.5. Mátrixok.

```
> with(linalg);
```

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*] (1.1.5.1)

```
> a:=matrix(2,3,[x,cc,Alma,3,5]);
```

$$a := \begin{bmatrix} x & cc & Alma \\ 3 & 5 & a_{2,\,3} \end{bmatrix} \tag{1.1.5.2}$$

```
> a[2,3]:=8;print(a);
```

$$a_{2,\,3} := 8$$

$$\begin{bmatrix} x & cc & Alma \\ 3 & 5 & 8 \end{bmatrix} \tag{1.1.5.3}$$

```
> b:=matrix([[x,y],[9,7]]);
```

$$b := \begin{bmatrix} x & y \\ 9 & 7 \end{bmatrix} \tag{1.1.5.4}$$

```
> c:=matrix(3,2,(i,j)->i+j-1);
```

$$c := \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix} \tag{1.1.5.5}$$

```
> d:=extend(c,1,1,0);copyinto(a,d,3,2);
```

$$d := \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & x & cc \\ 0 & 3 & 5 \end{bmatrix} \tag{1.1.5.6}$$

```
> transpose(c);d:=transpose(%);equal(c,d);
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

$$d := \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$$

$$true \tag{1.1.5.7}$$

```
> a:=matrix(3,2);
```

$$a := array(1..3, 1..2, [\,]) \tag{1.1.5.8}$$

```
> entermatrix(a);
Warning,  computation interrupted
```

1.1.6. Műveletek mátrixokkal.

```
> a:=matrix([[1,0],[0,2]]); b:=matrix([[0,1],[0,0]]);
  c:=matrix([[1,0],[0,1],[0,0]]);
```

$$a := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$b := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$c := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{1.1.6.1}$$

```
> d:=evalm(2*a+3*b); evalm(a&*b); evalm(b&*a); evalm(sin(d));
  evalm(d^2); evalm(c&*a); evalm(a&*c);
```

$$d := \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \sin(2) & \sin(3) \\ 0 & \sin(4) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 18 \\ 0 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

```
Error, (in linalg:-multiply) non matching dimensions for
vector/matrix product
```

▶ *1.1.7. Állítás.*

▼ *1.1.8. Nullmátrix, egységmátrix, inverz mátrix.*

```
> a:=matrix(3,2,0); iszero(a); b:=matrix(2,2,[1,0,0,0]);
  iszero(b);
```

$$a := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$true$$

$$b := \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$false \tag{1.1.8.1}$$

```
> rowdim(a); coldim(a);
```

$$3$$

$$2 \tag{1.1.8.2}$$

```
> a:=diag(3,4,5);
```

$$a := \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \tag{1.1.8.3}$$

```
> krondelta:=(i,j)-> if i=j then 1 else 0 fi;
```

$$krondelta := (i, j) \rightarrow \textbf{if } i = j \textbf{ then } 1 \textbf{ else } 0 \textbf{ end if} \tag{1.1.8.4}$$

```
> a:=matrix(3,3,krondelta);
```

$$a := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1.8.5}$$

```
> b:=matrix(3,2,krondelta);
```

$$b := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{1.1.8.6}$$

```
> c:=hilbert(3); d:=inverse(c); evalm(c&*d); evalm(d&*c);
```

$$c := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$d := \begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1.8.7}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1.8.7}$$

```
> inverse(b);
```

```
Error, (in linalg:-inverse) expecting a square matrix
```

```
> a:=matrix([[2,4],[1,2]]); inverse(a);
```

$$a := \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

```
Error, (in linalg:-inverse) singular matrix
```

1.1.9. Elemi sor- és oszlopműveletek.

```
> a:=matrix([[1,2,3],[4,5,6]]);
```

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \tag{1.1.9.1}$$

```
> swaprow(a,1,2);
```

$$\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \tag{1.1.9.2}$$

```
> swapcol(a,2,3);
```

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \end{bmatrix} \tag{1.1.9.3}$$

```
> addrow(a,1,2,-4);
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix} \tag{1.1.9.4}$$

```
> addcol(a,2,3,-3/2);
```

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & -\frac{3}{2} \end{bmatrix} \tag{1.1.9.5}$$

1.1.10. Gauss-féle kiküszöbölés.

```
> x:='x'; y:='y'; z:='z';
  eqns:={x+y/2+z/3=1,x/2+y/3+z/4=7/12,x/3+y/4+z/5=13/30};
```

$$x := x$$

$$y := y$$

$$z := z$$

$$eqns := \left\{ x + \frac{1}{2}\,y + \frac{1}{3}\,z = 1,\ \frac{1}{2}\,x + \frac{1}{3}\,y + \frac{1}{4}\,z = \frac{7}{12},\ \frac{1}{3}\,x + \frac{1}{4}\,y + \frac{1}{5}\,z = \frac{13}{30} \right\} \tag{1.1.10.1}$$

```
> solve(eqns);
```

$$\{z = 3,\ x = 1,\ y = -2\} \tag{1.1.10.2}$$

```
> a:=genmatrix(eqns,[x,y,z]);
```

$$a := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \tag{1.1.10.3}$$

```
> a:=genmatrix(eqns,[x,y,z],'b'); print(b);
```

$$a := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{7}{12} & \frac{13}{30} \end{bmatrix} \tag{1.1.10.4}$$

```
> geneqns(a,[x,y,z],b);
```

$$\left\{ x + \frac{1}{2}\,y + \frac{1}{3}\,z = 1,\ \frac{1}{2}\,x + \frac{1}{3}\,y + \frac{1}{4}\,z = \frac{7}{12},\ \frac{1}{3}\,x + \frac{1}{4}\,y + \frac{1}{5}\,z = \frac{13}{30} \right\} \tag{1.1.10.5}$$

```
> c:=augment(a,b);
```

$$c := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{13}{30} \end{bmatrix} \tag{1.1.10.6}$$

```
> genmatrix(eqns,[x,y,z],flag);
```

(1.1.10.7)

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{13}{30} \end{bmatrix} \tag{1.1.10.7}$$

1.1.11. Példák.

```
> c1:=pivot(c,1,1,2..3);
```

$$c1 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & \frac{1}{12} & \frac{4}{45} & \frac{1}{10} \end{bmatrix} \tag{1.1.11.1}$$

```
> c2:=pivot(c1,2,2,3..3);
```

$$c2 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{180} & \frac{1}{60} \end{bmatrix} \tag{1.1.11.2}$$

```
> gausselim(c,3);
```

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{180} & \frac{1}{60} \end{bmatrix} \tag{1.1.11.3}$$

```
> backsub(c2);
```

$$\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \tag{1.1.11.4}$$

```
> d:=copy(c); d[3,3]:=7/36; print(d);
```

$$d := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{13}{30} \end{bmatrix}$$

$$d_{3,3} := \frac{7}{36}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{7}{36} & \frac{13}{30} \end{bmatrix} \quad (1.1.11.5)$$

```
> d1:=pivot(d,1,1,2..3);
```

$$d1 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{10} \end{bmatrix} \quad (1.1.11.6)$$

```
> d2:=pivot(d1,2,2,3..3);
```

$$d2 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & 0 & \frac{1}{60} \end{bmatrix} \quad (1.1.11.7)$$

```
> backsub(d2);
Error, (in linalg:-backsub) inconsistent system
> e:=copy(d); e[3,4]:=5/12; print(e);
```

$$e := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{7}{36} & \frac{13}{30} \end{bmatrix}$$

$$e_{3,4} := \frac{5}{12}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{7}{36} & \frac{5}{12} \end{bmatrix} \tag{1.1.11.8}$$

```
> e1:=pivot(e,1,1,2..3);
```

$$e1 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} \tag{1.1.11.9}$$

```
> e2:=pivot(e1,2,2,3..3);
```

$$e2 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.1.11.10}$$

```
> backsub(e2);
```

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{6}\,_t_1 & 1 - _t_1 & _t_1 \end{bmatrix} \tag{1.1.11.11}$$

```
> evalm(randmatrix(3,4)+I*randmatrix(3,4)); gausselim(%);
  backsub(%);
```

$$\begin{bmatrix} 22+62\,I & -55-82\,I & -94+80\,I & 87-44\,I \\ -56+71\,I & -17\,I & -62-75\,I & 97-10\,I \\ -73-7\,I & -4-40\,I & -83+42\,I & -10-50\,I \end{bmatrix}$$

$$\left[\begin{array}{l}[-73-7\,I,-4-40\,I,-83+42\,I,-10-50\,I],[0,\frac{118682}{2689}+\frac{14957}{2689}\,I,\\ -\frac{269533}{5378}-\frac{1016897}{5378}\,I,\frac{418163}{2689}+\frac{35010}{2689}\,I],[0,0,\\ \frac{7738997}{36826}-\frac{8653977}{36826}\,I,\frac{1185088861}{5321357}+\frac{1475940899}{5321357}\,I]\end{array}\right.$$

$$\left[\begin{array}{l}[\frac{306082322429}{1057741824457}-\frac{35197922369}{1057741824457}\,I,\\ -\frac{1414384720421}{1057741824457}+\frac{989229441330}{1057741824457}\,I,\\ -\frac{195587870162}{1057741824457}+\frac{1177322214500}{1057741824457}\,I]\end{array}\right.$$ (1.1.11.12)

1.1.12. *Módosított Gauss-féle kiküszöbölés.*

```
> print(c);
```

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{7}{12} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{13}{30} \end{bmatrix}$$ (1.1.12.1)

```
> c1:=pivot(c,1,1,2..3);
```

$$c1 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & \frac{1}{12} & \frac{4}{45} & \frac{1}{10} \end{bmatrix}$$ (1.1.12.2)

```
> mulrow(c1,2,12); c2:=pivot(%,2,2,3..3);
```

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & \frac{1}{12} & \frac{4}{45} & \frac{1}{10} \end{bmatrix}$$

$$c2 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{1}{180} & \frac{1}{60} \end{bmatrix} \tag{1.1.12.3}$$

```
> c3:=mulrow(c2,3,180);
```

$$c3 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \tag{1.1.12.4}$$

```
> backsub(c3);
```

$$\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \tag{1.1.12.5}$$

1.1.13. Mátrixinverzió Gauss-féle kiküszöböléssel.

```
> a:=hilbert(3); b:=diag(1,1,1); c:=augment(a,b);
```

$$a := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$b := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(1.1.13.1)

$$c := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & 0 & 0 & 1 \end{bmatrix} \tag{1.1.13.1}$$

```
> cc:=gausselim(c,3);
```

$$cc := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & \frac{1}{12} & \frac{1}{12} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{180} & \frac{1}{6} & -1 & 1 \end{bmatrix} \tag{1.1.13.2}$$

```
> aa:=submatrix(cc,1..3,1..3); bb:=submatrix(cc,1..3,4..6);
```

$$aa := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{180} \end{bmatrix}$$

$$bb := \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{6} & -1 & 1 \end{bmatrix} \tag{1.1.13.3}$$

```
> backsub(aa,bb); inverse(a);
```

$$\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix} \tag{1.1.13.4}$$

```
> linsolve(a,b);
```

$$\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix} \tag{1.1.13.5}$$

1.1.14. Gauss–Jordan kiküszöbölés.

```
> print(c);
```

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & 0 & 0 & 1 \end{bmatrix} \tag{1.1.14.1}$$

```
> c1:=pivot(c,1,1);
```

$$c1 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & \frac{1}{12} & \frac{1}{12} & -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{12} & \frac{4}{45} & -\frac{1}{3} & 0 & 1 \end{bmatrix} \tag{1.1.14.2}$$

```
> mulrow(c1,2,12); c2:=pivot(%,2,2);
```

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & 1 & 1 & -6 & 12 & 0 \\ 0 & \frac{1}{12} & \frac{4}{45} & -\frac{1}{3} & 0 & 1 \end{bmatrix}$$

$$c2 := \begin{bmatrix} 1 & 0 & -\frac{1}{6} & 4 & -6 & 0 \\ 0 & 1 & 1 & -6 & 12 & 0 \\ 0 & 0 & \frac{1}{180} & \frac{1}{6} & -1 & 1 \end{bmatrix} \tag{1.1.14.3}$$

```
> mulrow(c2,3,180); c3:=pivot(%,3,3);
```

$$\begin{bmatrix} 1 & 0 & -\frac{1}{6} & 4 & -6 & 0 \\ 0 & 1 & 1 & -6 & 12 & 0 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{bmatrix}$$

$$c3 := \begin{bmatrix} 1 & 0 & 0 & 9 & -36 & 30 \\ 0 & 1 & 0 & -36 & 192 & -180 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{bmatrix} \quad (1.1.14.4)$$

```
> gaussjord(c,3);
```

$$\begin{bmatrix} 1 & 0 & 0 & 9 & -36 & 30 \\ 0 & 1 & 0 & -36 & 192 & -180 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{bmatrix} \quad (1.1.14.5)$$

*1.1.15. A műveletigények összehasonlítása.

*1.1.16. A háromszögmátrixok módszere.

```
> u:=LUdecomp(a,P='p',L='l');
```

$$u := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{180} \end{bmatrix} \quad (1.1.16.1)$$

```
> print(p); print(l); evalm(p&*l&*u);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 1 & 1 \end{bmatrix}$$

(1.1.16.2)

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \tag{1.1.16.2}$$

```
> forwardsub(l,b); backsub(u,%);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{6} & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix} \tag{1.1.16.3}$$

▶ *1.1.17. Numerikus megjegyzések.

▶ 1.1.18. Vektortér.

▼ 1.1.19. Példák.

```
> v:=vector(3,[5,6]);
```

$$v := [5 \quad 6 \quad v_3] \tag{1.1.19.1}$$

```
> v[3]:=7; print(v);
```

$$v_3 := 7$$

$$[5 \quad 6 \quad 7] \tag{1.1.19.2}$$

```
> u:=vector(3,i->i+1);
```

$$u := [2 \quad 3 \quad 4] \tag{1.1.19.3}$$

```
> evalm(2*u+v);
```

$$[9 \quad 12 \quad 15] \tag{1.1.19.4}$$

```
> w:=vector(4,0); vectdim(w);
```

$$w := [0 \quad 0 \quad 0 \quad 0]$$

$$4 \tag{1.1.19.5}$$

```
> w:=randvector(3);
```

$$w := [23 \quad 75 \quad -92] \tag{1.1.19.6}$$

```
> type(w,'vector'); type(w,'matrix');
```

$$true$$

$$\mathit{false} \tag{1.1.19.7}$$

1.1.20 Altér.

1.1.21. Példák.

1.1.22. Lineáris kombináció.

```
> evalm(-87*z-7/3*u+16/3*v);
```

$$\left[22-87\,z \quad 25-87\,z \quad 28-87\,z\right] \tag{1.1.22.1}$$

1.1.23. Megjegyzés.

```
> z:=vector([1,0,0]); a:=augment(z,u,v,w);
```

$$z := \left[1 \quad 0 \quad 0\right]$$

$$a := \begin{bmatrix} 1 & 2 & 5 & 23 \\ 0 & 3 & 6 & 75 \\ 0 & 4 & 7 & -92 \end{bmatrix} \tag{1.1.23.1}$$

```
> b:=stackmatrix(u,v,w,z);
```

$$b := \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 23 & 75 & -92 \\ 1 & 0 & 0 \end{bmatrix} \tag{1.1.23.2}$$

```
> col(a,2);
```

$$\left[2 \quad 3 \quad 4\right] \tag{1.1.23.3}$$

```
> col(a,2..3);
```

$$\left[2 \quad 3 \quad 4\right], \left[5 \quad 6 \quad 7\right] \tag{1.1.23.4}$$

```
> row(a,2);
```

$$\left[0 \quad 3 \quad 6 \quad 75\right] \tag{1.1.23.5}$$

```
> row(a,2..3);
```

$$\left[0 \quad 3 \quad 6 \quad 75\right], \left[0 \quad 4 \quad 7 \quad -92\right] \tag{1.1.23.6}$$

```
> subvector(b,3,2..3); subvector(b,1..3,2);
```

$$\left[75 \quad -92\right]$$

$$\left[3 \quad 6 \quad 75\right] \tag{1.1.23.7}$$

1.1.24. Tétel.

*1.1.25. Tétel.

1.1.26. Generátorrendszer, dimenzió.

1.1.27. Lineáris függetlenség és függőség.

▶ *1.1.28. Tétel.*

▼ *1.1.29. Bázis.*

```
> basis([z,u,v,w]); basis([u,v,w,z]);
```

$$[z, u, v]$$

$$[u, v, w] \tag{1.1.29.1}$$

```
> intbasis([z,u],[u,v,w]); intbasis([z,u],[v,w]); intbasis(
  [z,u],[v]);
```

$$\{[-937 \quad -1077 \quad -1436], [2 \quad 3 \quad 4]\}$$

$$\{[-937 \quad -1077 \quad -1436]\}$$

$$\{\ \}\tag{1.1.29.2}$$

```
> sumbasis([z,u],[v,w]);
```

$$[z, u, v] \tag{1.1.29.3}$$

▶ *1.1.30. Tétel.*

▶ *1.1.31. Következmény.*

▶ *1.1.32. Következmény.*

▶ *1.1.33. Következmény.*

▶ *1.1.34. Következmény.*

▶ *1.1.35. Következmény.*

▶ *1.1.36. Következmény.*

▶ *1.1.37. Következmény.*

▼ *1.1.38. Rang.*

```
> basis(a,colspace); basis(a,rowspace);
```

$$[[1 \quad 0 \quad 0], [2 \quad 3 \quad 4], [5 \quad 6 \quad 7]]$$

$$[[1 \quad 2 \quad 5 \quad 23], [0 \quad 3 \quad 6 \quad 75], [0 \quad 4 \quad 7 \quad -92]] \tag{1.1.38.1}$$

```
> print(a); backsub(gausselim(a));
```

$$\begin{bmatrix} 1 & 2 & 5 & 23 \\ 0 & 3 & 6 & 75 \\ 0 & 4 & 7 & -92 \end{bmatrix}$$

$$[-219 \quad -359 \quad 192] \tag{1.1.38.2}$$

▼ *1.1.39. Megjegyzés.*

```
> rank(a); gausselim(a,'ra');ra;
```

$$3$$

$$\begin{bmatrix} 1 & 2 & 5 & 23 \\ 0 & 3 & 6 & 75 \\ 0 & 0 & -1 & -192 \end{bmatrix}$$
$$3 \qquad (1.1.39.1)$$

```
> rank(b); gausselim(b,'rb');rb;
```

$$3$$
$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & -\frac{3}{2} & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$$3 \qquad (1.1.39.2)$$

▶ *1.1.40. Tétel.*
▶ *1.1.41. Következmény.*
▶ *1.1.42. Tétel.*
▶ *1.1.43. Koordináták.*
▶ *1.1.44. Izomorfia.*
▶ *1.1.45. Tétel.*
▶ **1.1.46. Vektorterek direkt összege.*
▶ **1.1.48. Következmény.*
▶ **1.1.49. Példa.*
▶ *1.1.50. Affin sokaságok.*
▼ *1.1.51. Tétel.*

```
> x:=vector([1+t,2-t,t-1]); x0:=map2(subs,t=0,x); y:=evalm(x-
  x0);
```

$$x := [1+t \quad 2-t \quad t-1]$$
$$x0 := [1 \quad 2 \quad -1]$$
$$y := [t \quad -t \quad t] \qquad (1.1.50.1)$$

```
> y1:=map2(subs,t=1,y);
```

$$y1 := [1 \quad -1 \quad 1] \qquad (1.1.50.2)$$

```
> z:=evalm(x0+t*y1);
```

$$z := [1+t \quad 2-t \quad t-1] \qquad (1.1.50.3)$$

▶ *1.1.52. Következmény.*

1.1.53. Következmény.

```
> x:=vector([1+t1,2-t2,t1+t2-1]); x0=map2(subs,[t1=0,t2=0],x)
  ;
  y:=evalm(x-x0);
```

$$x := [1 + t1 \quad 2 - t2 \quad t1 + t2 - 1]$$

$$x0 = [1 \quad 2 \quad -1]$$

$$y := [t1 \quad -t2 \quad t1 + t2] \tag{1.1.52.1}$$

```
> y1:=map2(subs,[t1=1,t2=0],y); y2:=map2(subs,[t1=0,t2=1],y);
```

$$y1 := [1 \quad 0 \quad 1]$$

$$y2 := [0 \quad -1 \quad 1] \tag{1.1.52.2}$$

```
> z:=evalm(x0+t1*y1+t2*y2);
```

$$z := [1 + t1 \quad 2 - t2 \quad t1 + t2 - 1] \tag{1.1.52.3}$$

```
> x:=vector([1+t1+t3,2+t2+t3,-1,2]); x0:=map2(subs,[t1=0,t2=
  0,t3=0],x);
  y:=evalm(x-x0);
```

$$x := [1 + t1 + t3 \quad 2 + t2 + t3 \quad -1 \quad 2]$$

$$x0 := [1 \quad 2 \quad -1 \quad 2]$$

$$y := [t1 + t3 \quad t2 + t3 \quad 0 \quad 0] \tag{1.1.52.4}$$

```
> y1:=map2(subs,[t1=1,t2=0,t3=0],y); y2:=map2(subs,[t1=0,t2=
  1,t3=0],y);
  y3:=map2(subs,[t1=0,t2=0,t3=1],y);
```

$$y1 := [1 \quad 0 \quad 0 \quad 0]$$

$$y2 := [0 \quad 1 \quad 0 \quad 0]$$

$$y3 := [1 \quad 1 \quad 0 \quad 0] \tag{1.1.52.5}$$

```
> basis([y1,y2,y3]);
```

$$[y1, y2] \tag{1.1.52.6}$$

```
> z:=evalm(x0+u1*y1+u2*y2);
```

$$z := [1 + u1 \quad 2 + u2 \quad -1 \quad 2] \tag{1.1.52.7}$$

1.1.54. Affin sokaság dimenziója.

1.2. Lineáris leképezések

```
> restart;with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, (1.2.1)

coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

▶ ***1.2.1. Lineáris leképezések.***

▶ ***1.2.2. Példák lineáris leképezésekre.***

▶ ***1.2.3. Tétel.***

▶ ***1.2.4. Lineáris leképezések összege és skalárszorosa.***

▶ ***1.2.5. Tétel.***

▶ ***1.2.6. Lineáris leképezések szorzata.***

▶ ***1.2.7. Tétel.***

▶ ***1.2.8. Következmény.***

▶ ***1.2.9. Tétel.***

▶ ***1.2.10. Következmény.***

▶ ***1.2.11. Következmény.***

▼ ***1.2.12. Vektorrendszer és leképezés mátrixa.***

```
> a:=matrix([[1,1,2,5,0],[0,1,1,3,-1],[1,0,1,2,1]]);
```

$$a := \begin{bmatrix} 1 & 1 & 2 & 5 & 0 \\ 0 & 1 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 & 1 \end{bmatrix} \qquad (1.2.12.1)$$

```
> colspace(a); rank(a); kernel(a); coldim(a);
```

$$\{[1 \quad 0 \quad 1], [0 \quad 1 \quad -1]\}$$

$$2$$

$$\{[-1 \quad -1 \quad 1 \quad 0 \quad 0], [-2 \quad -3 \quad 0 \quad 1 \quad 0], [-1 \quad 1 \quad 0 \quad 0 \quad 1]\}$$

$$5 \tag{1.2.12.2}$$

1.2.13. Tétel.

```
> a:=matrix([[1,2,3],[2,3,4],[3,4,5]]); x:=vector([1,0,1]);
  multiply(a,x);
```

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$x := [1 \quad 0 \quad 1]$$

$$[4 \quad 6 \quad 8] \tag{1.2.13.1}$$

1.2.14. Következmény.

1.2.15. Tétel.

1.2.16. Áttérésmátrix.

1.2.17. Példák: egyszerű bázistranszformációk.

1.2.18. Tétel.

1.2.19. Következmény.

1.2.20. Következmény.

1.2.21. Ekvivalens transzformációk, ekvivalens mátrixok.

```
> t:=matrix([[1,1,0],[0,1,1],[1,0,1]]); rank(t);
```

$$t := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$3 \tag{1.2.21.1}$$

```
> b:=evalm(inverse(t)&*a&*t);
```

$$b := \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{2} \\ 1 & \frac{1}{2} & \frac{3}{2} \\ 5 & \frac{9}{2} & \frac{11}{2} \end{bmatrix} \tag{1.2.21.2}$$

```
> s:=matrix([[1,-2,0],[0,1,-2],[-2,0,1]]); rank(s);
```

$$s := \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$3 \tag{1.2.21.3}$$

```
> evalm(inverse(s)&*evalm(a&*t));
```

$$\begin{bmatrix} -\frac{48}{7} & -\frac{41}{7} & -\frac{55}{7} \\ -\frac{38}{7} & -\frac{31}{7} & -\frac{45}{7} \\ -\frac{40}{7} & -\frac{33}{7} & -\frac{47}{7} \end{bmatrix} \tag{1.2.21.4}$$

```
> augment(s,evalm(a&*t)); gaussjord(%,3);
```

$$\begin{bmatrix} 1 & -2 & 0 & 4 & 3 & 5 \\ 0 & 1 & -2 & 6 & 5 & 7 \\ -2 & 0 & 1 & 8 & 7 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{48}{7} & -\frac{41}{7} & -\frac{55}{7} \\ 0 & 1 & 0 & -\frac{38}{7} & -\frac{31}{7} & -\frac{45}{7} \\ 0 & 0 & 1 & -\frac{40}{7} & -\frac{33}{7} & -\frac{47}{7} \end{bmatrix} \tag{1.2.21.5}$$

1.2.22. Lineáris formák.

```
> f:=matrix(1,4); x:=vector(4); multiply(f,x);
```

$$f := array(1..1, 1..4, [\,])$$

$$x := array(1..4, [\,])$$

$$[f_{1,1} x_1 + f_{1,2} x_2 + f_{1,3} x_3 + f_{1,4} x_4] \tag{1.2.22.1}$$

*1.2.23. Tétel.

1.2.24. Bilineáris leképezések.

```
> a:=matrix(2,3); x:=vector(2); y:=vector(3); innerprod(x,a,
  y);
  evalm(x&*a&*y);
```

$$a := array(1..2, 1..3, [\,])$$

$$x := array(1..2, [\,])$$

$$y := array(1..3, [\,])$$

$$y_1\,x_1\,a_{1,\,1} + y_1\,x_2\,a_{2,\,1} + y_2\,x_1\,a_{1,\,2} + y_2\,x_2\,a_{2,\,2} + y_3\,x_1\,a_{1,\,3} + y_3\,x_2\,a_{2,\,3}$$

$$\left(x_1\,a_{1,\,1} + x_2\,a_{2,\,1}\right) y_1 + \left(x_1\,a_{1,\,2} + x_2\,a_{2,\,2}\right) y_2 + \left(x_1\,a_{1,\,3} + x_2\,a_{2,\,3}\right) y_3 \qquad (1.2.24.1)$$

```
> x:=vector(3); innerprod(x,y);
```

$$x := array(1..3, [\,])$$

$$x_1\,y_1 + x_2\,y_2 + x_3\,y_3 \qquad (1.2.24.2)$$

1.2.25. Tétel.

1.2.26. Következmény.

1.2.27. Kvadratikus leképezések.

```
> b:=matrix([[1,2,3],[2,3,4],[5,6,7]]); q:=innerprod(x,b,x);
```

$$b := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

$$q := x_1^2 + 4\,x_1\,x_2 + 8\,x_1\,x_3 + 3\,x_2^2 + 10\,x_2\,x_3 + 7\,x_3^2 \qquad (1.2.27.1)$$

1.2.28. Tétel.

```
> s:=evalm(1/2*b+1/2*transpose(b)); a:=evalm(1/2*b-1/2*
  transpose(b));
```

$$s := \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$

$$a := \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \qquad (1.2.28.1)$$

```
> innerprod(x,s,x); innerprod(x,a,x);
```

$$x_1^2 + 4\,x_1\,x_2 + 8\,x_1\,x_3 + 3\,x_2^2 + 10\,x_2\,x_3 + 7\,x_3^2$$

$$0 \qquad (1.2.28.2)$$

1.2.29. Példa: vektori szorzat.

```
> a:=matrix([[0,k,-j],[-k,0,i],[j,-i,0]]); b:=innerprod(x,a,
  y);
```

$$a := \begin{bmatrix} 0 & k & -j \\ -k & 0 & i \\ j & -i & 0 \end{bmatrix}$$

$$b := -y_1 x_2 k + y_1 x_3 j + y_2 x_1 k - y_2 x_3 i - y_3 x_1 j + y_3 x_2 i \qquad (1.2.29.1)$$

```
> i:=vector([1,0,0]); j:=vector([0,1,0]); k:=vector([0,0,1]);
  evalm(b);
```

$$i := [1 \quad 0 \quad 0]$$
$$j := [0 \quad 1 \quad 0]$$
$$k := [0 \quad 0 \quad 1]$$
$$[-y_2 x_3 + y_3 x_2 \quad y_1 x_3 - y_3 x_1 \quad -y_1 x_2 + y_2 x_1] \qquad (1.2.29.2)$$

```
> crossprod(x,y);
```

$$[-y_2 x_3 + y_3 x_2 \quad y_1 x_3 - y_3 x_1 \quad -y_1 x_2 + y_2 x_1] \qquad (1.2.29.3)$$

1.2.30. Tétel: szimmetrikus és alternáló bilineáris leképezések mátrixa.

1.2.31. Szimmetrikus bilineáris formák átlós alakra hozása.

```
> print(s); addcol(s,1,2,-2); addrow(%,1,2,-2); addcol(%,1,3,
  -4);
  s1:=addrow(%,1,3,-4);
```

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 4 \\ 2 & -1 & 5 \\ 4 & -3 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & -1 & -3 \\ 4 & -3 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -3 \\ 4 & -3 & -9 \end{bmatrix}$$

(1.2.31.1)

$$s1 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -3 \\ 0 & -3 & -9 \end{bmatrix} \tag{1.2.31.1}$$

```
> addcol(s1,2,3,-3); s2:=addrow(%,2,3,-3);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$$

$$s2 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.2.31.2}$$

▶ *1.2.32. Megjegyzés.*

▶ **1.2.33. Antiszimmetrikus bilineáris formák átlós alakra hozása.*

▼ *1.2.34. Definit és indefinit kvadratikus formák.*

```
> definite(s,'positive_def'); definite(s,'positive_semidef');
  definite(s,'negative_semidef');
```

false

false

false (1.2.34.1)

```
> princaxis:=proc(b) local a,i,j,k,n,x;
  if not(type(b,'matrix'(rational,square))
    or type(b,'matrix'(float,square))) then ERROR(`Invalid
  argument`) fi;
  n:=coldim(b);
  for i to n do for j from i+1 to n do
    if b[i,j]<>b[j,i] then ERROR(`Invalid argument`) fi;
  od; od;
  a:=copy(b);
  for k to n do
    if a[k,k]=0 then
      for j from k+1 to n do if a[j,j]<>0 then break fi; od;
      if j<=n then a:=swapcol(a,k,j); a:=swaprow(a,k,j); fi;
    fi;
    if a[k,k]=0 then
      for j from k+1 to n do if a[k,j]<>0 then break fi; od;
      if j<=n then a:=addcol(a,j,k); a:=addrow(a,j,k); fi;
    fi;
    if a[k,k]<>0 then
```

```
      for j from k+1 to n do
        x:=-a[k,j]/a[k,k]; a:=addcol(a,k,j,x);a:=addrow(a,k,
    j,x);
      od;
    fi;
  od; evalm(a); end;
```

$$princaxis := \mathbf{proc}(b) \tag{1.2.34.2}$$

local a, i, j, k, n, x;

if not $(type(b, 'linalg:-matrix'(rational, square))$ **or** $type(b, 'linalg:-matrix'(float, square)))$ **then**

$ERROR(`Invalid\ argument`)$

end if;

$n := linalg:-coldim(b)$;

for i **to** n **do**

for j **from** $i+1$ **to** n **do**

if $b[i, j] <> b[j, i]$ **then**

$ERROR(`Invalid\ argument`)$

end if

end do

end do;

$a := copy(b)$;

for k **to** n **do**

if $a[k, k] = 0$ **then**

for j **from** $k+1$ **to** n **do**

if $a[j, j] <> 0$ **then**

break

end if

end do;

if $j <= n$ **then**

$a := linalg:-swapcol(a, k, j)$;

$a := linalg:-swaprow(a, k, j)$

end if

end if;

```
        if a[k, k] = 0 then
            for j from k + 1 to n do
                if a[k, j] <> 0 then
                    break
                end if
            end do;
            if j <= n then
                a := linalg:-addcol(a, j, k);
                a := linalg:-addrow(a, j, k)
            end if
        end if;
        if a[k, k] <> 0 then
            for j from k + 1 to n do
                x := -a[k, j] / a[k, k];
                a := linalg:-addcol(a, k, j, x);
                a := linalg:-addrow(a,
                k, j, x)
            end do
        end if
    end do;
    evalm(a)
end proc
```

```
> princaxis(s);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.2.34.3}$$

▶ *1.2.35. Sylvester–féle tehetetlenségi tétel.*

▶ **1.2.36. Konjugált bilineáris leképezések.*

▶ **1.2.37. Tétel.*

▶ **1.2.38. Következmény.*

▼ **1.2.39. Hermite–formák.*

▶ **1.2.40. Multilineáris leképezések.*

▶ *1.2.41. Tétel.

▶ *1.2.42. Példa.

▶ *1.2.43. Tenzorok, tenzorszorzat.

▶ *1.2.44. Segédtétel.

▶ *1.2.45. Tenzorok koordinátáinak transzformációja.

▶ *1.2.46. Einstein-konvenció.

▶ 1.2.47. Permutációk.

▶ 1.2.48. Tétel.

▶ 1.2.49. Következmény.

▶ 1.2.50. Terület, térfogat és determináns.

▶ 1.2.51. Tétel.

▶ 1.2.52. Következmény.

▶ *1.2.53. Következmény.

▶ 1.2.54. Következmény.

▼ 1.2.55. Mátrix determinánsa.

```
> a:=matrix(4,4); det(a);
```

$$a := array(1..4, 1..4, [\,])$$

$$\begin{aligned}
&a_{1,1}\,a_{2,2}\,a_{3,3}\,a_{4,4} - a_{1,1}\,a_{2,2}\,a_{3,4}\,a_{4,3} - a_{1,1}\,a_{3,2}\,a_{2,3}\,a_{4,4} \\
&\quad + a_{1,1}\,a_{3,2}\,a_{2,4}\,a_{4,3} \\
&\quad + a_{1,1}\,a_{4,2}\,a_{2,3}\,a_{3,4} - a_{1,1}\,a_{4,2}\,a_{2,4}\,a_{3,3} - a_{2,1}\,a_{1,2}\,a_{3,3}\,a_{4,4} \\
&\quad + a_{2,1}\,a_{1,2}\,a_{3,4}\,a_{4,3} \\
&\quad + a_{2,1}\,a_{3,2}\,a_{1,3}\,a_{4,4} - a_{2,1}\,a_{3,2}\,a_{1,4}\,a_{4,3} - a_{2,1}\,a_{4,2}\,a_{1,3}\,a_{3,4} \\
&\quad + a_{2,1}\,a_{4,2}\,a_{1,4}\,a_{3,3} \\
&\quad + a_{3,1}\,a_{1,2}\,a_{2,3}\,a_{4,4} - a_{3,1}\,a_{1,2}\,a_{2,4}\,a_{4,3} - a_{3,1}\,a_{2,2}\,a_{1,3}\,a_{4,4} \\
&\quad + a_{3,1}\,a_{2,2}\,a_{1,4}\,a_{4,3} \\
&\quad + a_{3,1}\,a_{4,2}\,a_{1,3}\,a_{2,4} - a_{3,1}\,a_{4,2}\,a_{1,4}\,a_{2,3} - a_{4,1}\,a_{1,2}\,a_{2,3}\,a_{3,4} \\
&\quad + a_{4,1}\,a_{1,2}\,a_{2,4}\,a_{3,3} \\
&\quad + a_{4,1}\,a_{2,2}\,a_{1,3}\,a_{3,4} - a_{4,1}\,a_{2,2}\,a_{1,4}\,a_{3,3} - a_{4,1}\,a_{3,2}\,a_{1,3}\,a_{2,4} \\
&\quad + a_{4,1}\,a_{3,2}\,a_{1,4}\,a_{2,3}
\end{aligned} \tag{1.2.55.1}$$

▼ 1.2.56. Tétel.

```
> a:=matrix(1,1); det(a);
```

$$a := array(1..1, 1..1, [\,])$$

$$a_{1,\,1} \tag{1.2.56.1}$$

```
> a:=matrix(2,2); det(a);
```

$$a := array(1..2, 1..2, [\,])$$

$$a_{1,\,1}\,a_{2,\,2} - a_{1,\,2}\,a_{2,\,1} \tag{1.2.56.2}$$

```
> a:=matrix(3,3); det(a);
```

$$a := array(1..3, 1..3, [\,])$$

$$a_{1,\,1}\,a_{2,\,2}\,a_{3,\,3} - a_{1,\,1}\,a_{2,\,3}\,a_{3,\,2} - a_{2,\,1}\,a_{1,\,2}\,a_{3,\,3} + a_{2,\,1}\,a_{1,\,3}\,a_{3,\,2} + a_{3,\,1}\,a_{1,\,2}\,a_{2,\,3} - a_{3,\,1}\,a_{1,\,3}\,a_{2,\,2} \tag{1.2.56.3}$$

▶ *1.2.57. Tétel.*

▶ *1.2.58. Következmény.*

▶ *1.2.59. Részmátrix, aldeterminánsok.*

▶ *1.2.60. Tétel.*

▶ *1.2.61. Tétel.*

▼ *1.2.62. Tétel.*

```
> a:=hilbert(3); det(a); gausselim(a,'ra','da'); da;
```

$$a := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$\frac{1}{2160}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & \frac{1}{180} \end{bmatrix}$$

$$\frac{1}{2160} \tag{1.2.62.1}$$

▼ *1.2.63. Kifejtési tétel.*

```
> delcols(a,2..3);delrows(a,1..1);
```

$$\begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \tag{1.2.63.1}$$

```
> delcols(delrows(a,1..1),1..1);A11:=minor(a,1,1);A12:=minor
  (a,1,2);A13:=minor(a,1,3);
```

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$A11 := \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$A12 := \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{5} \end{bmatrix}$$

$$A13 := \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix} \tag{1.2.63.2}$$

```
> a[1,1]*det(A11)-a[1,2]*det(A12)+a[1,3]*det(A13);
```

$$\frac{1}{2160} \tag{1.2.63.3}$$

▶ *1.2.64. Lineáris transzformáció determinánsa.

▶ *1.2.65. Determinánsok szorzástétele.

▶ 1.2.66. Determinánsok szorzástétele.

▶ 1.2.67. Megjegyzés: lineáris transzformáció determinánsa.

▶ *1.2.68. Tétel.*

▼ *1.2.69. Tétel.*

```
> b:=adjoint(a); evalm(a&*b);
```

$$b := \begin{bmatrix} \frac{1}{240} & -\frac{1}{60} & \frac{1}{72} \\ -\frac{1}{60} & \frac{4}{45} & -\frac{1}{12} \\ \frac{1}{72} & -\frac{1}{12} & \frac{1}{12} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2160} & 0 & 0 \\ 0 & \frac{1}{2160} & 0 \\ 0 & 0 & \frac{1}{2160} \end{bmatrix} \tag{1.2.69.1}$$

▼ *1.2.70. Lineáris egyenletek.*

```
> a:=matrix([[1,1,2,5,0],[0,1,1,3,-1],[1,0,1,2,1]]);
```

$$a := \begin{bmatrix} 1 & 1 & 2 & 5 & 0 \\ 0 & 1 & 1 & 3 & -1 \\ 1 & 0 & 1 & 2 & 1 \end{bmatrix} \tag{1.2.70.1}$$

```
> colspace(a); rank(a); kernel(a); coldim(a);
```

$$\{[1 \quad 0 \quad 1], [0 \quad 1 \quad -1]\}$$

$$2$$

$$\{[-1 \quad -1 \quad 1 \quad 0 \quad 0], [-2 \quad -3 \quad 0 \quad 1 \quad 0], [-1 \quad 1 \quad 0 \quad 0 \quad 1]\}$$

$$5 \tag{1.2.70.2}$$

```
> b:=vector([0,0,0]); linsolve(a,b);
```

$$b := [0 \quad 0 \quad 0]$$

$$[-_t_1 - 2\,_t_2 - _t_3 \quad -_t_1 - 3\,_t_2 + _t_3 \quad _t_1 \quad _t_2 \quad _t_3] \tag{1.2.70.3}$$

```
> b:=vector([1,1,1]); linsolve(a,b);
```

$$b := [1 \quad 1 \quad 1] \tag{1.2.70.4}$$

▶ *1.2.71. Állítás.*

▶ *1.2.72. A szuperpozíció elve.*

▶ *1.2.73. Következmény.*

▶ *1.2.74. Következmény.*

▶ *1.2.75. Következmény.*

▶ *1.2.76. Következmény.*

▼ *1.2.77. Következmény.*

```
> b:=vector([0,0,0]); linsolve(a,b);
```

$$b := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -_t_1 - 2\,_t_2 - _t_3 & -_t_1 - 3\,_t_2 + _t_3 & _t_1 & _t_2 & _t_3 \end{bmatrix} \tag{1.2.77.1}$$

```
> b:=vector([1,1,0]); linsolve(a,b);
```

$$b := \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -_t_2 - 2\,_t_1 - _t_3 & 1 - _t_2 - 3\,_t_1 + _t_3 & _t_2 & _t_1 & _t_3 \end{bmatrix} \tag{1.2.77.2}$$

▶ *1.2.78. Invariáns alterek.*

▶ *1.2.79. Sajátérték, sajátvektor, sajátaltér.*

▼ *1.2.80. Karakterisztikus polinom.*

```
> a:=matrix([[1,2,I],[2,-I,3],[-1,2*I,4]]);
  evalm(-charmat(a,lambda)); det(%); (-1)^3*charpoly(a,
  lambda);
```

$$a := \begin{bmatrix} 1 & 2 & I \\ 2 & -I & 3 \\ -1 & 2\,I & 4 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda + 1 & 2 & I \\ 2 & -\lambda - I & 3 \\ -1 & 2\,I & -\lambda + 4 \end{bmatrix}$$

$$-\lambda^3 + 5\,\lambda^2 - I\,\lambda^2 + 10\,I\,\lambda - 25 - 10\,I$$

$$-\lambda^3 + 5\,\lambda^2 - I\,\lambda^2 + 10\,I\,\lambda - 25 - 10\,I \tag{1.2.80.1}$$

▼ *1.2.81. Tétel.*

```
> evalf(solve(%,lambda)); %[1]+%[2]+%[3]; trace(a);
```

$$4.395490370 + 1.351578558\,I,\ 2.434682355 - 1.229735917\,I,$$

$$-1.830172723 - 1.121842643\,I$$

$$5.000000002 - 1.000000002\,I$$

$$5 - I \tag{1.2.81.1}$$

```
> evalf(eigenvals(a));
```

$$4.395490370 + 1.351578558\,I,\ 2.434682355 - 1.229735917\,I,\ -1.830172723 - 1.121842643\,I \tag{1.2.81.2}$$

```
> evalf(eigenvects(a));
```

$$[4.395490370 + 1.351578558\,I,\ 1.,\ \{[[1.,\ 1.954505136 - 0.6951699149\,I,\ 2.741918387 + 0.513519901\,I]]\}],\ [2.434682355 - 1.229735917\,I,\ 1.,\ \{[[1.,\ 0.5074154780 - 0.4694586252\,I,\ -0.2908186661 - 0.4198513982\,I]]\}],\ [-1.830172723 - 1.121842643\,I,\ 1.,\ \{[[1.,\ -1.211920613 - 0.5853714612\,I,\ 0.0489002809 + 0.4063314964\,I]]\}] \tag{1.2.81.3}$$

1.2.82. Algebrai multiplicitás.

```
> matrix([[1,1,0],[0,1,0],[0,0,5]]); eigenvects(%);
```

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$[5,\ 1,\ \{[0\quad 0\quad 1]\}],\ [1,\ 2,\ \{[1\quad 0\quad 0]\}] \tag{1.2.82.1}$$

```
> a:=matrix([[1,1,0],[0,1,0],[0,0,1]]); eigenvects(%);
```

$$a := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[1,\ 3,\ \{[1\quad 0\quad 0],\ [0\quad 0\quad 1]\}] \tag{1.2.82.2}$$

```
> matrix([[1,1,0],[0,1,1],[0,0,1]]); eigenvects(%);
```

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[1,\ 3,\ \{[1\quad 0\quad 0]\}] \tag{1.2.82.3}$$

*1.2.83. Cayley–Hamilton-tétel.

```
> a:=matrix([[1,0,0],[1,1,0],[0,0,2]]); p:=charpoly(a,lambda)
  ; expand(p);
```

```
L:=PolynomialTools[CoefficientList](p,lambda);
L:=zip((x,y)->x*y,L,[a&^i$i=0..nops(L)-1]);
p:=convert(L,`+`); evalm(p);
```

$$a := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$p := (\lambda - 1)^2 (\lambda - 2)$$

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2$$

$$L := [-2, 5, -4, 1]$$

$$L := [-2\ a\ \&\hat{}\ 0, 5\ a\ \&\hat{}\ 1, -4\ a\ \&\hat{}\ 2, a\ \&\hat{}\ 3]$$

$$p := -2\ a\ \&\hat{}\ 0 + 5\ a\ \&\hat{}\ 1 - 4\ a\ \&\hat{}\ 2 + a\ \&\hat{}\ 3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.2.83.1}$$

1.2.84. Felső háromszög alak.

```
> t:=matrix([[1,0,0],[0,1,2],[0,1,1]]); tm:=inverse(t); b:=
  evalm(tm&*a&*t);
```

$$t := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$tm := \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$b := \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 2 \\ 1 & -1 & 0 \end{bmatrix} \tag{1.2.84.1}$$

```
> eigenvects(b);
```

$$[1, 2, \{[0 \quad -1 \quad 1]\}], [2, 1, \{[0 \quad -2 \quad 1]\}] \tag{1.2.84.2}$$

```
> f11:=vector([0,-2,1]); e11:=vector([1,0,0]); e12:=vector(
  [0,1,0]); e13:=vector([0,0,1]); basis([f11,e11,e12,e13]);
```

$$f11 := [0 \quad -2 \quad 1]$$

$$e11 := [1 \quad 0 \quad 0]$$

$$e12 := \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$e13 := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$[f11, e11, e12] \tag{1.2.84.3}$$

```
> t1:=matrix([[0,1,0],[-2,0,1],[1,0,0]]); t1m:=inverse(t1);
  bb:=evalm(t1m&*b&*t1);
```

$$t1 := \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$t1m := \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$bb := \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \tag{1.2.84.4}$$

```
> b1:=matrix([[1,0],[1,1]]); eigenvects(b1);
```

$$b1 := \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$[1, 2, \{[0 \quad 1]\}] \tag{1.2.84.5}$$

```
> f22:=vector([0,1]); e22:=vector([1,0]); e23:=vector([0,1]);
  basis([f22,e22,e23]);
```

$$f22 := [0 \quad 1]$$

$$e22 := [1 \quad 0]$$

$$e23 := [0 \quad 1]$$

$$[f22, e22] \tag{1.2.84.6}$$

```
> t2:=matrix([[0,1],[1,0]]); t2m:=inverse(t2); bb1:=evalm
  (t2m&*b1&*t2);
```

$$t2 := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$t2m := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$bb1 := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{1.2.84.7}$$

```
> b2:=matrix([[1]]); eigenvects(b2);
```

$$b2 := [1]$$

$$[1, 1, \{[1]\}] \tag{1.2.84.8}$$

```
> f23:=1*e22; print(f23);
```

$$f23 := e22$$

$$[1 \quad 0] \tag{1.2.84.9}$$

```
> print(f11); f12:=0*e11+1*e12; print(f12); f13:=1*e11+0*e12;
  print(f13);
```

$$[0 \quad -2 \quad 1]$$

$$f12 := e12$$

$$[0 \quad 1 \quad 0]$$

$$f13 := e11$$

$$[1 \quad 0 \quad 0] \tag{1.2.84.10}$$

```
> t:=matrix([[0,0,1],[-2,1,0],[1,0,0]]); tm:=inverse(t);
  print(evalm(tm&*b&*t));
```

$$t := \begin{bmatrix} 0 & 0 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$tm := \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.2.84.11}$$

▶ *1.2.85. Következmény.*

▶ *1.2.86. Segédtétel.*

▼ *1.2.87. Átlós alak.*

```
> a:=matrix([[2,0,0],[0,1,0],[0,0,3]]);
  t:=matrix([[1,-4,3],[1,-2,2],[3,-3,-2]]); tm:=inverse(t);
  b:=evalm(tm&*a&*t);
```

$$a := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$t := \begin{bmatrix} 1 & -4 & 3 \\ 1 & -2 & 2 \\ 3 & -3 & -2 \end{bmatrix}$$

$$tm := \begin{bmatrix} -\frac{10}{13} & \frac{17}{13} & \frac{2}{13} \\ -\frac{8}{13} & \frac{11}{13} & -\frac{1}{13} \\ -\frac{3}{13} & \frac{9}{13} & -\frac{2}{13} \end{bmatrix}$$

$$b := \begin{bmatrix} \frac{15}{13} & \frac{28}{13} & -\frac{38}{13} \\ -\frac{14}{13} & \frac{51}{13} & -\frac{20}{13} \\ -\frac{15}{13} & \frac{24}{13} & \frac{12}{13} \end{bmatrix} \qquad (1.2.87.1)$$

```
> L:=eigenvects(b); b1:=op(1,L[1][3]); b2:=op(1,L[2][3]);
  b3:=op(1,L[3][3]);
  tb:=augment(b1,b2,b3); evalm(inverse(tb)&*b&*tb);
```

$$L := \left[3, 1, \left\{\left[-2 \quad 1 \quad 2\right]\right\}\right], \left[2, 1, \left\{\left[\frac{10}{3} \quad \frac{8}{3} \quad 1\right]\right\}\right], \left[1, 1, \left\{\left[\frac{17}{9} \quad \frac{11}{9} \quad 1\right]\right\}\right]$$

$$b1 := \left[-2 \quad 1 \quad 2\right]$$

$$b2 := \left[\frac{10}{3} \quad \frac{8}{3} \quad 1\right]$$

$$b3 := \left[\frac{17}{9} \quad \frac{11}{9} \quad 1\right]$$

$$tb := \begin{bmatrix} -2 & \frac{10}{3} & \frac{17}{9} \\ 1 & \frac{8}{3} & \frac{11}{9} \\ 2 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1.2.87.2)$$

1.3. Belső szorzat

```
> restart;with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] (1.3.1)

1.3.1. Norma.

1.3.2. Példák.

```
> x:=vector([1,1,1]);
  norm(x,1), norm(x,1.5), norm(x,2), norm(x,3), norm(x,
  infinity), norm(x);
  evalf(%);
```

$$x := [1 \quad 1 \quad 1]$$

$$3,\ 2.080083823,\ \sqrt{3},\ 3^{1/3},\ 1,\ 1$$

$$3.,\ 2.080083823,\ 1.732050808,\ 1.442249570,\ 1.,\ 1. \qquad (1.3.2.1)$$

```
> y:=vector([1,2,3+4*I]);
  norm(y,1), norm(y,1.5), norm(y,2), norm(y,3), norm(y,
  infinity), norm(y);evalf(%);
```

$$y := [1 \quad 2 \quad 3+4\,I]$$

$$8,\ 6.084571667,\ \sqrt{30},\ 134^{1/3},\ 5,\ 5$$

$$8.,\ 6.084571667,\ 5.477225575,\ 5.117229947,\ 5.,\ 5. \qquad (1.3.2.2)$$

▶ *1.3.3. Belső szorzat.*

▶ *1.3.4. Tétel.*

▼ *1.3.5. Példa.*

```
> dotprod(x,y); dotprod(y,x);
```

$$6-4\,I$$

$$6+4\,I \qquad (1.3.5.1)$$

▶ *1.3.6. Megjegyzés.*

▼ *1.3.7. Szögek, euklidészi tér, unitér tér.*

```
> y:=vector([1,-1,0]); angle(x,y); angle(y,x);
```

$$y := [1 \quad -1 \quad 0]$$

$$\frac{1}{2}\,\pi$$

$$\frac{1}{2}\,\pi \qquad (1.3.7.1)$$

▼ *1.3.8. Definíció.*

```
> z:=vector([2,3,4]); dotprod(x,y); dotprod(y,z);
```

$$z := [2 \quad 3 \quad 4]$$

$$0$$

$$-1 \qquad (1.3.8.1)$$

▼ *1.3.9. Vegyes szorzat.*

```
> dotprod(x,crossprod(y,z)); det(augment(x,y,z));
```

$$-3$$

$$-3 \qquad (1.3.9.1)$$

▶ *1.3.10. Általánosított Pythagorasz-tétel.*

▶ *1.3.11. Következmény.*

▼ *1.3.12. Gram-Schmidt-ortogonalizálás.*

```
> basis([z,y,x]);
```

$$[z, y, x] \tag{1.3.12.1}$$

```
> GramSchmidt([z,y,x]);
```

$$\left[[2, 3, 4], \left[\frac{31}{29}, -\frac{26}{29}, \frac{4}{29}\right], \left[\frac{4}{19}, \frac{4}{19}, -\frac{5}{19}\right]\right] \tag{1.3.12.2}$$

```
> L:=map(normalize,%);
```

$$L := \left[\left[\frac{2}{29}\sqrt{29} \quad \frac{3}{29}\sqrt{29} \quad \frac{4}{29}\sqrt{29}\right], \tag{1.3.12.3}$$

$$\left[\frac{31}{1653}\sqrt{57}\sqrt{29} \quad -\frac{26}{1653}\sqrt{57}\sqrt{29} \quad \frac{4}{1653}\sqrt{57}\sqrt{29}\right],$$

$$\left[\frac{4}{57}\sqrt{3}\sqrt{19} \quad \frac{4}{57}\sqrt{3}\sqrt{19} \quad -\frac{5}{57}\sqrt{3}\sqrt{19}\right]\Bigg]$$

```
> map(norm,L,2);
```

$$[1, 1, 1] \tag{1.3.12.4}$$

▶ *1.3.13. Következmény.*

▶ *1.3.14. Következmény.*

▶ *1.3.15. Tétel.*

▶ *1.3.16. Az approximáció alapfeladata normált térben.*

▶ *1.3.17. Legjobb lineáris approximáció belső szorzat térben.*

▶ *1.3.18. Következmény: Bessel–egyenlőtlenség.*

▶ *1.3.19. Ortogonális felbontási tétel.*

▶ *1.3.20. Következmény.*

▶ *1.3.21. Következmény.*

▶ *1.3.22. Előállítási tétel.*

▶ *1.3.23. Adjungált leképezés.*

▶ *1.3.24. Tétel.*

▼ *1.3.25. Adjungált operátor mátrixa.*

```
> a:=matrix([[1,2,I],[2,-I,3]]); htranspose(a);
```

$$a := \begin{bmatrix} 1 & 2 & \mathrm{I} \\ 2 & -\mathrm{I} & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & \mathrm{I} \\ -\mathrm{I} & 3 \end{bmatrix} \tag{1.3.25.1}$$

```
> d:=matrix(L); ds:=htranspose(d);
```

$$d := \begin{bmatrix} \frac{2}{29}\sqrt{29} & \frac{3}{29}\sqrt{29} & \frac{4}{29}\sqrt{29} \\ \frac{31}{1653}\sqrt{57}\sqrt{29} & -\frac{26}{1653}\sqrt{57}\sqrt{29} & \frac{4}{1653}\sqrt{57}\sqrt{29} \\ \frac{4}{57}\sqrt{3}\sqrt{19} & \frac{4}{57}\sqrt{3}\sqrt{19} & -\frac{5}{57}\sqrt{3}\sqrt{19} \end{bmatrix}$$

$$ds := \begin{bmatrix} \frac{2}{29}\sqrt{29} & \frac{31}{1653}\sqrt{57}\sqrt{29} & \frac{4}{57}\sqrt{3}\sqrt{19} \\ \frac{3}{29}\sqrt{29} & -\frac{26}{1653}\sqrt{57}\sqrt{29} & \frac{4}{57}\sqrt{3}\sqrt{19} \\ \frac{4}{29}\sqrt{29} & \frac{4}{1653}\sqrt{57}\sqrt{29} & -\frac{5}{57}\sqrt{3}\sqrt{19} \end{bmatrix} \qquad (1.3.25.2)$$

```
> evalm(d&*ds); evalm(ds&*d);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1.3.25.3)$$

1.3.26. Önadjungált és normális transzformációk.

```
> b:=matrix([[1,2,I],[2,3,-3*I],[-I,3*I,-1]]); bs:=htranspose
  (b);
```

$$b := \begin{bmatrix} 1 & 2 & \mathrm{I} \\ 2 & 3 & -3\,\mathrm{I} \\ -\mathrm{I} & 3\,\mathrm{I} & -1 \end{bmatrix}$$

$$bs := \begin{bmatrix} 1 & 2 & \mathrm{I} \\ 2 & 3 & -3\,\mathrm{I} \\ -\mathrm{I} & 3\,\mathrm{I} & -1 \end{bmatrix} \qquad (1.3.26.1)$$

```
> evalm(b&*bs); evalm(bs&*b);
```

$$\begin{bmatrix} 6 & 5 & -6\,\mathrm{I} \\ 5 & 22 & -4\,\mathrm{I} \\ 6\,\mathrm{I} & 4\,\mathrm{I} & 11 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 & -6\,\mathrm{I} \\ 5 & 22 & -4\,\mathrm{I} \\ 6\,\mathrm{I} & 4\,\mathrm{I} & 11 \end{bmatrix} \tag{1.3.26.2}$$

```
> c:=matrix([[I,2,1],[-2,3*I,4],[-1,-4,0]]); cs:=htranspose
  (c);
```

$$c := \begin{bmatrix} \mathrm{I} & 2 & 1 \\ -2 & 3\,\mathrm{I} & 4 \\ -1 & -4 & 0 \end{bmatrix}$$

$$cs := \begin{bmatrix} -\mathrm{I} & -2 & -1 \\ 2 & -3\,\mathrm{I} & -4 \\ 1 & 4 & 0 \end{bmatrix} \tag{1.3.26.3}$$

```
> evalm(c&*cs); evalm(cs&*c);
```

$$\begin{bmatrix} 6 & 4-8\,\mathrm{I} & -8-\mathrm{I} \\ 4+8\,\mathrm{I} & 29 & 2-12\,\mathrm{I} \\ -8+\mathrm{I} & 2+12\,\mathrm{I} & 17 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4-8\,\mathrm{I} & -8-\mathrm{I} \\ 4+8\,\mathrm{I} & 29 & 2-12\,\mathrm{I} \\ -8+\mathrm{I} & 2+12\,\mathrm{I} & 17 \end{bmatrix} \tag{1.3.26.4}$$

1.3.27. Példa.

```
> a:=matrix([[1,2,I],[2,-I,3],[-1,2*I,4]]); as:=htranspose(a)
  ;
```

$$a := \begin{bmatrix} 1 & 2 & \mathrm{I} \\ 2 & -\mathrm{I} & 3 \\ -1 & 2\,\mathrm{I} & 4 \end{bmatrix}$$

$$as := \begin{bmatrix} 1 & 2 & -1 \\ 2 & \mathrm{I} & -2\,\mathrm{I} \\ -\mathrm{I} & 3 & 4 \end{bmatrix} \tag{1.3.27.1}$$

```
> evalm(a&*as); evalm(as&*a);
```

$$\begin{bmatrix} 6 & 2+5\,I & -1 \\ 2-5\,I & 14 & 8 \\ -1 & 8 & 21 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2-4\,I & 2+I \\ 2+4\,I & 9 & -3\,I \\ 2-I & 3\,I & 26 \end{bmatrix} \qquad (1.3.27.2)$$

▶ ***1.3.28.Polarizációs formula.***

▶ ***1.3.29. Következmény.***

▶ ***1.3.30. Következmény.***

▶ ***1.3.31. Tétel.***

▼ ***1.3.32. Megjegyzés.***

```
> orthog(d); orthog(ds);
```

true

true (1.3.32.1)

▶ ***1.3.33. Tétel.***

▼ ***1.3.34. Felső háromszög alak.***

```
> a:=map(evalf,a); as:=map(evalf,as);
  eigenvects(as); e3:=normalize(op(1,%[1][3]));
```

$$a := \begin{bmatrix} 1. & 2. & 1.\,I \\ 2. & -1.\,I & 3. \\ -1. & 2.\,I & 4. \end{bmatrix}$$

$$as := \begin{bmatrix} 1. & 2. & -1. \\ 2. & 1.\,I & -2.\,I \\ -1.\,I & 3. & 4. \end{bmatrix}$$

$$[4.395490368 - 1.351578556\,I,\ 1,\ \{[[0.2014302949 - 0.1949564944\,I,\ 0.4867552051 - 0.0799409069\,I,\ 0.5530548001 + 0.7743399524\,I]]\}],\ [-1.830172721 + 1.121842640\,I,\ 1,\ \{[[-0.4108131960 + 0.4175601040\,I,$$

$$0.2289381566 - 0.6993571032\,I,$$

$$-0.2363592574 + 0.2439207700\,I]]\}], [2.434682347$$

$$+ 1.229735916\,I, 1,$$

$$\{[[-0.1293138255 - 0.8134486314\,I,$$

$$0.2451412887 - 0.4247678157\,I,$$

$$-0.3245201583 + 0.4765266198\,I]]\}]$$

$$e3 := [[0.1818176709 - 0.1759742037\,I, \quad (1.3.34.1)$$

$$0.4393614066 - 0.07215731631\,I,$$

$$0.4992056219 + 0.6989449463\,I]]$$

```
> f11:=vector([1,0,0]); c1:=dotprod(f11,e3);
  g11:=normalize(evalm(f11-c1*e3));
```

$$f11 := [1 \quad 0 \quad 0]$$

$$c1 := 0.1818176709 + 0.1759742037\,I$$

$$g11 := [[0.9674582239 + 0.\,I, -0.09569559874 - 0.06635614535\,I, \quad (1.3.34.2)$$

$$0.03331604002 - 0.2221572455\,I]]$$

```
> f12:=vector([0,1,0]); c1:=dotprod(f12,e3); c2:=dotprod(f12,
  g11);
  g12:=normalize(evalm(f12-c1*e3-c2*g11));
```

$$f12 := [0 \quad 1 \quad 0]$$

$$c1 := 0.4393614066 + 0.07215731631\,I$$

$$c2 := -0.09569559874 + 0.06635614535\,I$$

$$g12 := [[9.011008512\,10^{-11} - 6.758256384\,10^{-11}\,I, \quad (1.3.34.3)$$

$$0.8878029573 + 0.\,I, -0.2032556789 - 0.4129080271\,I]]$$

```
> a1:=matrix(2,2):a1[1,1]:=dotprod(multiply(a,g11),g11):
  a1[2,1]:=dotprod(multiply(a,g11),g12):
  a1[1,2]:=dotprod(multiply(a,g12),g11):
  a1[2,2]:=dotprod(multiply(a,g12),g12):
  print(a1);a1s:=htranspose(a1);
```

$$\begin{bmatrix} 1.031800907 - 0.1027395788\,I & 2.262127593 - 0.2100924609\,I \\ 2.336176755 - 0.5768639631\,I & -0.4272912741 - 2.248838979\,I \end{bmatrix}$$

$$a1s := \begin{bmatrix} [1.031800907 + 0.1027395788\,I, & (1.3.34.4) \\ 2.336176755 + 0.5768639631\,I], [2.262127593 + 0.2100924609\,I, & \end{bmatrix}$$

$$-0.4272912741 + 2.248838979\,I]\Big]$$

```
> eigenvects(a1s); f2:=op(1,%[1][3]);
  e2:=normalize(evalm(f2[1]*g11+f2[2]*g12));
```

$$[2.434682356 + 1.229735916\,I,\ 1,\ \{[[0.7609893873 - 0.2494771120\,I,\ 0.5947216369 + 0.07044499461\,I]]\},\ [-1.830172722 + 1.121842644\,I,\ 1,\ \{[[-0.6831686965 - 0.1713046639\,I,\ 0.8387464071 - 0.2952655257\,I]]\}]$$

$$f2 := [[0.7609893873 - 0.2494771120\,I,\ 0.5947216369 + 0.07044499461\,I]]$$

$$e2 := [[0.7362254411 - 0.2413586837\,I,\ 0.4386179534 + 0.03591881375\,I,\ -0.1218632415 - 0.4372545786\,I]] \qquad (1.3.34.5)$$

```
> f21:=vector([1,0,0]); c1:=dotprod(f21,e2); c2:=dotprod(f21,
  e3);
  e1:=normalize(evalm(f21-c1*e2-c2*e3));
```

$$f21 := [1\quad 0\quad 0]$$

$$c1 := 0.7362254411 + 0.2413586837\,I$$

$$c2 := 0.1818176709 + 0.1759742037\,I$$

$$e1 := [[0.5793906279 + 0.\,I,\ -0.7021754454 - 0.3391587390\,I,\ 0.02833236381 + 0.2354246616\,I]] \qquad (1.3.34.6)$$

```
> ta:=augment(e1,e2,e3); evalm(inverse(ta)&*a&*ta);
```

$$ta := \Big[[0.5793906279 + 0.\,I,\ 0.7362254411 - 0.2413586837\,I,\ 0.1818176709 - 0.1759742037\,I],\ [-0.7021754454 - 0.3391587390\,I,\ 0.4386179534 + 0.03591881375\,I,\ 0.4393614066 - 0.07215731631\,I],\ [0.02833236381 + 0.2354246616\,I,\ -0.1218632415 - 0.4372545786\,I,$$

$$0.4992056219 + 0.6989449463\,I]\Bigg]$$

$$\Bigg[[-1.830172720 - 1.121842644\,I,\ 0.9579862903 + 2.073920836\,I, \tag{1.3.34.7}$$

$$-0.5275243096 - 0.5584367480\,I],\ [-3.8\,10^{-9} - 1.2\,10^{-9}\,I,$$

$$2.434682352 - 1.229735912\,I,\ -0.8677528658 + 1.114851675\,I],$$

$$[1.3\,10^{-9} + 4.\,10^{-10}\,I,\ 1.\,10^{-9} - 2.\,10^{-9}\,I,$$

$$4.395490368 + 1.351578556\,I]\Bigg]$$

▶ ***1.3.35. Segédtétel.***

▼ ***1.3.36. Normális transzformáció átlós alakja.***

```
> print(b); Lb:=evalf(eigenvects(b));
```

$$\begin{bmatrix} 1 & 2 & I \\ 2 & 3 & -3\,I \\ -I & 3\,I & -1 \end{bmatrix}$$

$$Lb := \Big[5.064969131 - 1.\,10^{-9}\,I,\ 1., \tag{1.3.36.1}$$

$$\{[[1.,\ 2.590807476 - 7.958787212\,10^{-10}\,I,$$

$$8.976397949\,10^{-10} + 1.116645820\,I]]\},\ \Big[-3.400399149$$

$$- 5.660254040\,10^{-10}\,I,\ 1.,$$

$$\{[[1.,\ -1.225863560 - 7.92192835\,10^{-11}\,I,$$

$$-6.193515427\,10^{-10} + 1.948672028\,I]]\},\ \Big[1.335430019$$

$$+ 1.166025404\,10^{-9}\,I,\ 1.,$$

$$\{[[1.,\ -0.1296497970 + 4.888495928\,10^{-10}\,I,$$

$$-2.34384875\,10^{-11} - 0.5947296133\,I]]\}\Big]$$

```
> b1:=op(1,Lb[1][3]); b2:=op(1,Lb[2][3]); b3:=op(1,Lb[3][3]);
  tb:=augment(normalize(b1),normalize(b2),normalize(b3));
```

```
evalm(inverse(tb)&*b&*tb);
```

$$b1 := \left[[1., 2.590807476 - 7.958787212\ 10^{-10}\ I,\ 8.976397949\ 10^{-10} + 1.116645820\ I]\right]$$

$$b2 := \left[[1., -1.225863560 - 7.92192835\ 10^{-11}\ I,\ -6.193515427\ 10^{-10} + 1.948672028\ I]\right]$$

$$b3 := \left[[1., -0.1296497970 + 4.888495928\ 10^{-10}\ I,\ -2.34384875\ 10^{-11} - 0.5947296133\ I]\right]$$

$$tb := \left[\begin{array}{l} [0.3340918169, 0.3984075083, 0.8541979365], [\\ 0.8655675769 - 2.658965680\ 10^{-10}\ I, \\ -0.4883932465 - 3.156155735\ 10^{-11}\ I, \\ -0.1107465891 + 4.175743134\ 10^{-10}\ I], [\\ 2.998941100\ 10^{-10} + 0.3730622308\ I, \\ -2.467543049\ 10^{-10} + 0.7763655672\ I, \\ -2.002110766\ 10^{-11} - 0.5080168085\ I] \end{array}\right]$$

$$\left[\begin{array}{l} [5.064969132 + 0.\ I, -2.\ 10^{-9} + 5.480145087\ 10^{-10}\ I, \\ -3.\ 10^{-10} + 1.375856464\ 10^{-9}\ I], [5.\ 10^{-10} + 8.716398007\ 10^{-10}\ I, \\ -3.400399150 - 6.\ 10^{-19}\ I, 1.\ 10^{-9} + 8.922149470\ 10^{-10}\ I], [\\ 1.3\ 10^{-9} - 6.780241944\ 10^{-10}\ I, 1.1\ 10^{-9} - 5.771081405\ 10^{-10}\ I, \\ 1.335430019 + 5.\ 10^{-19}\ I] \end{array}\right] \qquad (1.3.36.2)$$

```
> print(c); evalf(eigenvects(c));
```

$$\begin{bmatrix} I & 2 & 1 \\ -2 & 3\,I & 4 \\ -1 & -4 & 0 \end{bmatrix}$$

$$\begin{aligned}&\left[-2.\,10^{-9}+0.9126830441\,I,\ 1.,\right.\\&\quad\{[[-3.704757828+1.690634827\,I,\ 1.,\\&\quad -1.852378915+0.3234881743\,I]]\},\ [1.913397460\,10^{-9}\\&\quad +6.360361017\,I,\ 1.,\\&\quad\{[[0.120868449-0.3843834841\,I,\ 1.,\\&\quad 0.060434224+0.6478985117\,I]]\},\ [2.086602540\,10^{-9}\\&\quad -3.273044063\,I,\ 1.,\\&\quad\{[[0.308027319+0.5040934836\,I,\ 1.,\\&\quad 0.1540136595-1.316214273\,I]]\}\left.\right]\end{aligned}\tag{1.3.36.3}$$

```
> map(evalf,d); Ld:=eigenvects(%);
  abs(Ld[1][1]); abs(Ld[2][1]); abs(Ld[3][1]);
```

$$\begin{bmatrix} 0.3713906763 & 0.5570860147 & 0.7427813528 \\ 0.7624744001 & -.6394946585 & 0.09838379358 \\ 0.5298129430 & 0.5298129430 & -.6622661788 \end{bmatrix}$$

$$\begin{aligned}Ld:=&\left[-0.9651850825+0.2615678890\,I,\ 1,\right.\\&\quad\{[[0.4651221388-0.4407628439\,I,\\&\quad -1.029175978-0.1082692855\,I,\\&\quad 0.09014493297+1.038110678\,I]]\},\ [0.999999998,\ 1,\\&\quad\{[-.890383449\quad -.4395241902\quad -.4238805372]\},\\&\quad[-0.9651850825-0.2615678890\,I,\ 1,\\&\quad\{[[0.4651221388+0.4407628439\,I,\\&\quad -1.029175978+0.1082692855\,I,\\&\quad 0.09014493297-1.038110678\,I]]\}\left.\right]\end{aligned}$$

$$\begin{aligned}&1.000000002\\&0.999999998\\&1.000000002\end{aligned}\tag{1.3.36.4}$$

▶ ***1.3.37. Következmény.***

▶ ***1.3.38. Szimmetrikus transzformáció átlós alakja.***

► 2. Többváltozós függvények

► 3. Függvénysorozatok és függvenysorok