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Contents

IMPERFECTION-SENSITIVITY AND CATASTROPHE THEORY

Zsolt Gáspár

An elastic structure is in equilibrium position if the gradient of its total potential energy function is equal to zero. The equilibrium position is critical if the Hessian matrix of the energy function is singular. Let us suppose a one-parameter load, and the imperfections of the structure are prescribed by some imperfection parameters. Imperfection-sensitivity curves/surfaces show the critical load parameter against the imperfection parameter(s).

Koiter [1] has shown how the critical load parameter depends on the imperfection for the most common cases (limit point, asymmetric, stable and unstable symmetric point of bifurcation). Later Thompson and Hunt [2] analyzed the imperfection-sensitivity surfaces for the semi-symmetric cases (monoclinal, anticlinal and homeoclinal point of bifurcation) where two critical points coincide. These results were achieved before the catastrophe theory was developed.

Thom's theorem classifies the points of typical r-parameter family of smooth functions $R^n \to R$, for any n and for $r \leq 5$. Using its results imperfection-sensitivities of other types of catastrophes (e.g. butterfly, parabolic and symbolic umbilic) were analyzed [3, 4].

Unfortunately if the structure has two symmetry planes and the designer made an optimization to have a double critical point, usually double cusp catastrophes occur which does not appear in Thom's theorem. These cases will be dealt with in this lecture. There are 15 classes and many subclasses of double cusp catastrophes. Determinacy of the energy function is usually four, but it is higher in some classes. The effect of imperfections disturbing linear terms of the total potential energy function are considered. Generally the change of the critical load is proportional with the 2/3 power of the absolute value of the imperfection, but in special cases other exponents are obtained.

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EMPIRICAL PORTFOLIO STRATEGIES

László Györfi

For some assets, the log-optimal investment strategy maximizes the conditional expectation of the log-return given the past data. The rule assumes the knowledge of the process' distributions. In this talk we present empirical investment strategies, which depend on the data only, and have optimal asymptotic growth rate. The performance of these strategies measured on NYSE data is solid, and sometimes even spectacular.

Critical phenomena in portfolio selection

Imre Kondor

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Rational portfolio selection is seeking a tradeoff between risk and expected return, by optimizing the risk functional over the portfolio weights, given the expected return and possibly other constraints. In real life the risk functional is not given in advance, but has to be estimated from observations on the market. Since the size N of banking portfolios (i.e. the dimension of space in which the minimum is sought) is large and the number of observed data (the lengths T of the time series for the various assets) is always bounded, we never have a sufficient amount of information to reliably reconstruct the underlying stochastic process and estimate the risk functional. Our estimates of risk will therefore fluctuate from sample to sample, and the weights of the optimal portfolio will fluctuate along with them. This problem has long been known in finance, and a large number of noise reduction techniques have been developed to deal with it over the past decades, at least in the simple case when the risk functional is chosen to be the variance. Variance is, however, not the only risk measure in use, and in some cases (e.g. for fat tailed distributions) it can even be grossly misleading. Noise reduction techniques for alternative risk measures are either much less developed or nonexisting.

Applying a variety oftecniques (simulations, random matrix theory, replica method) we have performed a comparative study of the noise sensitivity of various risk measures (variance, mean absolute deviation, expected shortfall, and maximal loss) recently. We have found that the noise sensitivity strongly depends on the ratio N/T, and that different risk measures exhibit very different sensitivity to the same noise. We have also observed that there exists a feasibility boundary, a critical value of N/T, for all these risk measures, beyond which the risk functional becomes unbounded, hence the optimization meaningless. This critical ratio is 1 for the variance and mean absolute deviation, ½ for maximal loss, and a value, smaller than ½ (and depending on the threshold beyond which the conditional average loss is calculated) for expected shortfall. Upon approaching this critical point the fluctuations in the estimation error of risk increase tremendously: the average error diverges, so does also the variance of its distribution. The weights of the optimal portfolio for a given sample also show strong deviations from their ideal values, with the variance and all the higher moments of their distribution diverging as one approaches the critical point.

These critical indices associated with these divergences seem to be universal, i.e. independent of the structure of the market, whereas the prefactors of the scaling laws do depend on the covariance structure (predominantly positive correlations enhancing, negative ones decreasing the strength of the divergence).

When short selling is excluded (or any other constraint is applied that makes the domain over which the optimum is sought finite) sample to sample fluctuations can obviously not diverge. In these cases the instability of portfolio selection manifests itself through some or most of the weights sticking to the boundaries defined by the constraints. In the case of a ban on short selling this leads to a spontaneous reduction of the portfolio size. Clearly, in these cases the solution is determined more by the constraints than by the objective function.

These observations are highly relevant for portfolio selection, especially in the cases where no efficient filtering techniques are known.

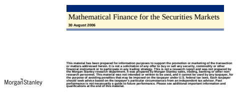
The observed critical phenomena in portfolio selection ares related to (and represent a simple class of) the critical phenomena discovered in complex optimization problems recently.

This work has been supported by the "Cooperative Center for Communication Networks Data Analysis", a NAP project sponsored by the National Office of Research and Technology under grant No.\ KCKHA005.

MATHEMATICAL FINANCE FOR THE SECURITIES MARKETS

Joseph Langsam

This talk is a brief survey of the class of problems in mathematical finance that are of current interest for an international securities firm.



Click here the view the presentation.

MATHEMATICAL MODELING OF MOLECULAR REGULATORY NETWORKS

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The ultimate goal of molecular cell biology is to understand the physiology of living cells in terms of the information that is encoded in the genome of the cell. According to the central dogma of molecular biology, the nucleotide sequences of DNA are translated into amino-acid sequences, and the linear sequence of amino acids directs the folding of a protein into its native three-dimensional shape. That shape, in turn, determines the functional properties of the protein. Individual proteins carry out their function in complex networks of interacting macromolecules, the coordinated behavior of which determines the physiological properties of the living cell. These networks of interacting proteins are intrinsically dynamic: they describe how a cell changes in space and time to respond to stimuli, grow and reproduce, differentiate, and do all the other remarkable tricks that are necessary to stay alive and perpetuate the species. These networks receive information from outside and inside the cell by signal-transduction pathways, process the information to make ‘decisions’, then trigger responses that are appropriate to the survival and reproduction of the cell. Therefore these networks can be considered the ‘brain’ of the cell. In this lecture, I will show few representative quantitative examples to derive the physiological properties of a cell from the wiring diagrams of its control systems.

FINDING SENSE IN LARGE SETS OF DATA – SOME POWERFUL APPLICATIONS OF MATHEMATICS AND COMPUTER SCIENCE

Lajos Rónyai – András Benczúr, Jr.

The January 23, 2006 issue of Business Week carried the cover story "Math will rock your world". It gives an account of successful new businesses, which use substantially mathematics and computing in their operation. The talk will touch upon some of these new phenomena. By a subjective selection of examples we intend to demonstrate that mathematical and computational ideas can be used very efficiently in a wide range of problem domains, including data mining, document handling, web search, computational genomics and cryptography.

Some related efforts and results from MTA SZTAKI and her partners will also be outlined.

THE EVOLUTION OF TIME DELAY MODELS FOR HIGH PERFORMANCE MANUFACTURING

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F. W. Taylor, the 3rd president of the American Society of Mechanical Engineers stated in his paper 'On the Art of Cutting Metals' in 1907 that

"… Chatter is the most obscure and delicate of all problems facing the machinist – probably no rules or formulae can be devised which will accurately guide the machinist in taking maximum cuts and speeds possible without producing chatter."

In this context, chatter means the high-frequency machine tool vibrations. Indeed, vibrations arising during metal cutting still cause serious problems in manufacturing technology; they reduce efficiency, damage machine tool parts, and affect the surface quality of the cut in a negative way.

The mathematical modelling of machine tool vibrations is linked to the development of the theory of delay differential equations, more exactly, to that of the (retarded) differential-difference equations (DDE). As the first stability results appeared for the autonomous DDEs, the development of the manufacturing technology was directed to the increase of the cutting speed. These results together with the development of the numerically controlled (NC) machine tools led to the advanced cutting technology called 'high-speed milling' in the 1990's. The new and peculiar vibration phenomena of high-speed milling required the application of non-autonomous DDEs as mathematical models. Recently, the extreme increase of the feed rate provides a new direction in metal cutting, and the study of the so-called 'high-performance cutting' requires the use of models that include state-dependent time delays.

In this presentation, the parallel development of the mathematical theory and the manufacturing technology will be reviewed.

CONSTRAINED SPHERICAL CIRCLE PACKINGS

Tibor Tarnai

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in collaboration with

Patrick Fowler

University of Sheffield, UK

Many situations in the natural sciences and technological applications can be modelled as variants of the packing problem - the determination of efficient packings of objects in an appropriate space. In one version of the packing problem, the task is to arrange n equal circles (spherical caps) without overlap on a sphere, so that their angular radius r is a maximum. In applications it can be more useful to deal with a constrained version of this spherical circle packing problem. For example, the arrangement of packed circles may be required to exhibit a particular overall point-group symmetry; or a given number of circles form a morphological unit, and identical copies of these units should be packed on a sphere. A survey of families of multisymmetric packings, in tetrahedral, octahedral and icosahedral groups will be presented, and the polymorphism of these packings will be analysed. Additionally, three problems will be studied here: How must kN non-overlapping equal circles forming N units be packed on a sphere so that the angular diameter of the circles will be as large as possible under the constraint that, within each unit, the k circle centres lie (1) for k = 2, at the end points of a diameter; (2) for k = 3, at the vertices of an equilateral triangle inscribed in a great circle; (3) for k = 4, at the vertices of a regular tetrahedron. Computer-generated putative solutions have been calculated and will be presented for different values of N. Finally, for k = 2, the problem of packing of twin circles is investigated, where a twin is defined as two circles that are constrained to touch. Here solutions to the constrained problem are mainly expected as perfect matchings in the graphs of the solutions to the respective unconstrained problem.