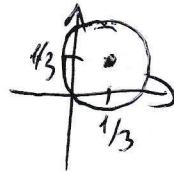


Pötzh. megoldás, A csapat

① $|z+1| = |2z-i|$. Legyen $z=x+yi$, $x, y \in \mathbb{R}$. $|x+yi+1| = |2x+2yi-i| \Leftrightarrow (x+1)^2 + y^2 = 4x^2 + (2y-1)^2$

 $\Leftrightarrow 0 = 3x^2 - 2x + 3y^2 - 2y \Leftrightarrow 0 = x^2 - \frac{2}{3}x + y^2 - \frac{2}{3}y \Leftrightarrow \left(x - \frac{1}{3}\right)^2 + \left(y - \frac{1}{3}\right)^2 = \frac{2}{9}$. A negoldás eges $\frac{1}{3} + \frac{1}{3}i$ középpontú, $\frac{\sqrt{2}}{3}$ sugarú körönél pontjai.



② $\frac{\bar{z}}{z} + \frac{z}{\bar{z}} = 1 \Leftrightarrow (\bar{z})^2 + z^2 - z\bar{z} = 0$. Ha $z = a+bi$, $a, b \in \mathbb{R}$, akkor elől $2a^2 + 2b^2 - (a^2 + b^2) = 0 \Leftrightarrow a^2 = b^2 \Leftrightarrow a = \pm b$, ahol $z \neq 0$ miatt $b \neq 0$. Tehát a megoldások: $z = \pm \sqrt{b} + bi$, $b \neq 0$.

③ $n=1$ esetén $\sum_{k=1}^1 \frac{1}{(2k-1)(2k+1)} = \frac{1}{(2-1)(2+1)} = \frac{1}{3} = \frac{1}{2 \cdot 1 + 1}$

Teh. $\sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$. Belátható, hogy azaz $\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} = \frac{n+1}{2n+3}$.

$$\begin{aligned} \sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} &= \sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2n+1)(2n+3)} = \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} = \frac{n(2n+3)+1}{(2n+1)(2n+3)} = \\ &= \frac{2n^2+3n+1}{(2n+1)(2n+3)} = \frac{(n+1)(2n+1)}{(2n+1)(2n+3)} = \frac{n+1}{2n+3} \end{aligned}$$

④ a) $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+1}}{3\sqrt{3n^2+2n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{1}{n^2}}}{3\sqrt{3+\frac{2}{n}}} = \frac{1}{3\sqrt{3}}$

b) $\lim_{x \rightarrow 3} \frac{x^2-x-6}{2x^2-x-15} = \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{(x-3)(2x+5)} = \lim_{x \rightarrow 3} \frac{x+2}{2x+5} = \frac{5}{11}$

c) $\lim_{x \rightarrow 0} \frac{tg^2 x - \sin^2 x}{x^4} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^4} = \lim_{x \rightarrow 0} \frac{\sin x \left(\frac{1 - \cos x}{\cos x}\right)}{\cos^3 x \cdot x^4} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^4 \frac{1}{\cos^3 x} \approx 1$

⑤ $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ divergens a ~~minimálisan~~ elenél, mert $\frac{n}{2n^2+1} \geq \frac{n}{3n^2} = \frac{1}{3n}$, és

$\sum_{n=1}^{\infty} \frac{1}{n}$ divergens. Mivelent $\lim_{n \rightarrow \infty} \frac{n}{2n^2+1} = \lim_{n \rightarrow \infty} \frac{1}{2n + \frac{1}{n}} = 0$, $\frac{n+1}{2(n+1)^2+1} - \frac{n}{2n^2+1} = \frac{(n+1)(2n^2+1) - n(2(n+1)^2+1)}{(2(n+1)^2+1)(2n^2+1)} = \frac{-2n^2-2n+1}{(2(n+1)^2+1)(2n^2+1)} < 0 \Rightarrow \{\frac{n}{2n^2+1}\}$ monoton csökken

$(-1)^n \frac{n}{2n^2+1}$ váltakozó előjelű. Ezért $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n^2+1}$ Leibniz típusú sor és fellebbelesen konvergens.