

Exam topics for Stochastic processes

2022/23 autumn semester

1 Theoretical questions

1. Definition of Markov chains, Markov property, gambler's ruin, Ehrenfest chain
2. Stationary distribution, method to compute, explicitly for two states, multistep transition probabilities, $\lim_{n \rightarrow \infty} P^n$
3. Classification of states, irreducibility, period, recurrence, transience, convergence theorems (without proofs)
4. Irreducible and primitive matrices, Perron–Frobenius theorem and its application to Markov chains, convergence for periodic Markov chains
5. Inventory chain, long-term profit, Wright–Fisher model without and with mutations, simple random walk on graphs, knight on the chessboard
6. Mean first passage time matrix (one step argument), stationary distribution of the Ehrenfest chain (generating functions)
7. Detailed balance, reversible Markov chains, simple random walk on graphs, time reversal, stationary distribution in birth and death chains
8. Exit distribution by one step reasoning, two year collage, Wright–Fisher model, gambler's ruin (fair and unfair), transience of unfair gambler's ruin
9. Mean exit time with one step argument, two year collage, tennis, gambler's ruin (fair and unfair), fundamental matrix
10. Generating functions of random variables, branching processes, probability of extinction
11. Strong Markov property, return times, recurrent and transient states, expected number of visits, simple random walk on \mathbb{Z}
12. Poisson process, interarrival times, distribution of points conditionally given their number in an interval, thinning, superposition, Poisson race
13. Barbershop example, continuous time Markov chains, infinitesimal generator, Kolmogorov's forward and backward equations
14. Stationary distribution of continuous time Markov chains, irreducibility and convergence, detailed balance
15. Construction of finite state Markov chains with routing matrix, birth and death chains with stationary distribution
16. $M/M/s$ queueing, stationary distribution for $M/M/\infty$, branching processes, explosion in finite time, Kolmogorov's three series theorem
17. Exit distribution and expected time of exit, when can the kindergarden teacher go home, $M/M/1$ queue with finite waiting room
18. Conditional expectation, definition, existence, properties
19. Martingales, definition, examples, variance formula for martingales in L^2
20. Betting strategies, profit process, stopping times, stopped (sub/super)martingale
21. Optional stopping theorem and its application to gambler's ruin
22. Convergence theorem for martingales, Pólya's urn
23. Wald's equality, Hoeffding's inequality
24. Multivariate normal distribution and its linear transformations
25. Brownian motion, definition, construction and properties

2 Exercises

1. 1.1
2. 1.3
3. 1.6
4. 1.7
5. 1.8
6. 1.9 b)
7. 1.11 c)
8. 1.13
9. 1.14
10. 1.15
11. 1.21
12. 1.26
13. 1.31
14. 1.36
15. 1.37
16. 1.43
17. 1.46
18. 1.48
19. 1.51
20. 1.63
21. 1.65
22. 1.67
23. 1.68
24. 1.70
25. 1.72
26. 1.73
27. 1.74
28. 1.77 (4.A)
29. 4.B
30. 4.C
31. 4.D
32. 4.E
33. 4.F
34. 2.1
35. 2.5
36. 2.6
37. 2.10
38. 2.16
39. 2.17
40. 2.22
41. 2.27
42. 2.29
43. 2.30
44. 2.31
45. 2.32
46. 2.33
47. 2.43
48. 2.46
49. 2.60
50. 4.2
51. 4.3
52. 4.8
53. 4.10
54. 4.12 (class)
55. 4.14

- 56. 4.19
- 57. 4.22
- 58. 8.A
- 59. 8.B
- 60. 8.C
- 61. 8.D
- 62. 8.E
- 63. 8.F
- 64. 5.2
- 65. 5.3
- 66. 5.6
- 67. 5.7
- 68. 5.8
- 69. 5.9
- 70. 5.10
- 71. 5.11 (class)
- 72. 10.A
- 73. 10.B
- 74. 10.C
- 75. 10.D
- 76. 10.E
- 77. 10.F
- 78. 10.G