

Probability 1 – Exercises

Tutorial no. 11

23rd Nov 2023

- 11.1** We throw a coin 10 times, with a probability p of heads and probability $(1 - p)$ of tails at each throw. Let X be the number of *pure runs* (e.g. in TTHTT, $X = 3$). Calculate $\mathbb{E}(X)$ and $\text{Var}(X)$. (Hint: write X as a sum of some indicator variables of some pretty simple events.)
- 11.2** We throw with a die n times. Let X be the number of times we throw 1, and Y be the number of times we throw 2. What is the correlation of the two random variables?
- HW 11.3** (3 points) There are 37 pockets on the roulette wheel, from 0 to 36. Xavier always bets that the result is at least 19. Yvette always bets that the result is $1 \pmod 3$ (so, 1, 4, 7, \dots , 34). Let's spin the wheel 20 times, independently. Let X be the number of times Xavier wins, and Y be the number of times Yvette wins. (It's possible that they both win a round, or that neither wins a round). What is the correlation of X and Y ?
- HW 11.4** (3 points) In an urn, there are M red and N blue balls. We take out all balls without replacement. Let X be the number of *pure runs*. (Similarly to question 11.1, but with red and blue instead of heads and tails). Calculate $\mathbb{E}(X)$.
- 11.5** Twelve people get on an elevator. They all choose a destination independently out of the 10 possible floors. Determine the expected value and the variance of the number of floors where the elevator stops.
- 11.6** Let (X, Y) has jointly uniform distribution on the triangle determined by the points $(-1, 0)$, $(0, 0)$, $(0, 2)$. (a) What is the two dimensional covariance matrix of (X, Y) ?
(b) Let $Z = 7X + 2Y$. What is the two dimensional covariance matrix of (X, Z) ?
- HW 11.7** (4 points) Let X and Y be random variables that can only take two values: $X \in \{x_1, x_2\}$ and $Y \in \{y_1, y_2\}$. Prove that if $\text{Cov}(X, Y) = 0$, then they are independent. (Not true in general!)
- 11.8** Let (X, Y) be the coordinates of a uniformly chosen point on the circumference of the circle centered at $(1, 1)$ with radius 1. $\text{Cov}(X, Y) = ?$