Probability 1 - Exercises

Tutorial no. 6 19th Oct 2023

HW 6.1 (2 points) Leo, when he goes hiking, has a small, equal and independent probability to fall with each step and hit his knee, or fall and hit his elbow. On a 10 kilometer hike, on average, he hits his knee 3 times, and his elbow twice. At most, how long of a hike can his mother let him go on if we want a $\frac{2}{3}$ probability for him to not hit his knee or elbow?

6.2 The police frequently check the speed of cars on the highway ("radaring"). Experience shows, that the probability that in 5 minutes, there will be a car that goes over the speed limit, is the same as the probability that there will be none.

(a) What is the probability that after 20 minutes of radaring, (i) exactly 2, (ii) at least 2 speeding cars will be caught?

(b) How long should the police radar for, if they want to catch at least 1 speeding car with a 95% probability.

6.3 Let's assume that the number of gold ores in a given area has $Poi(10)$ distribution. Each gold ore, independently of the others, is discovered with probability $\frac{1}{50}$. Calculate the probability that

(a) exactly 1,

(b) at least 1, and

(c) at most 1 ore is discovered this time.

Hint: First show that if a random variable has $Poi(\lambda)$ distribution, and each event is counted with p probability, independently of each other, then the number of events counted is $Poi(\lambda \cdot p)$.

HW 6.4 (2 points) The average density of trees in a forest is 16 trees in $100m^2$. The trunk of the trees are perfect cylinders, with a diameter of $20cm$ each. We shoot a gun $120m$ away from the edge of the forest, without aiming in the direction of the edge of the forest. What is the probability that we hit a tree? (Note: Disregard the fact that the centre of two trees can't be closer than 20cm to each other).

6.5 Odysseus wants to go home to Ithaca, but he angered Poseidon, who put the ocean full of whirlpools. The whirlpools are circular, and their density is 2 whirlpools every 10 square kilometers. If a boat gets closer than $150m$ to a whirlpool, it sinks. Otherwise, it can go around it. Odysseus doesn't know where the whirlpools are, he just goes straight to Ithaca. He studied Probability 1 at BME, so he knows that he has a 1% probability to make it home safely. How far away is Odysseus from Ithaca?

6.6 Milka introduced a new type of chocolate bar that contains raisins and peanuts. Each chocolate bar consists of 15 squares. Each chocolate bar has 30 raisins on average, and the probability of it having 0 peanuts is $\frac{1}{2}$.

(a) What is the distribution of the peanuts in the chocolate bar?

(b) We break off two squares. What is the probability that the number of peanuts and raising combined is less than 2?

6.7 We throw with a fair die until each number from 1 to 6 has occurred at least once. What is the expected value of the number of throws we need for this?

6.8 There are 4 red and 4 blue balls in an urn. We pick 4 balls at random out of the urn. If we picked 2 red and 2 blue, we stop, otherwise we put them back. We continue until we manage to pull out exactly 2 red and 2 blue. What is the probability that we pull exactly n times?

6.9 Amy and Bernadette are playing the following game. Amy throws with a fair die. She does not tell Bernadette, which throw she managed to land on 6 first, but she tells her which throw she managed to land on 6 the second time. After this, Bernadette needs to guess when the first 6 could have been. What should she guess? If she's smart, with what probability can she guess correctly?

♣ **6.10** (3 points) Let X be a Poisson distribution with parameter λ. Show that

$$\mathbb{P}(X \text{is even}) = \frac{1}{2}(1 + \exp(-2\lambda)))$$

6.11 Andrew and Bella are playing the following game. They throw 2 fair dice, and Andrew pays Bella the same amount of forint as the square of the difference of the two numbers, while Bella pays Andrew the sum of the two numbers. Who is favoured in this game?

HW 6.12 (1+2+1 points) Two people are competing in archery. The contestants have a p_1 and p_2 probability to hit the target ($p_1 < p_2$) with each shot. The person who is less skilled starts, and then they alternate. The first person to hit the target wins.

(a) What is the probability that the more skilled person wins?

(b) What is the expected time of the game, if there is one shot every minute?

(c) Give a simple expression for the expected time if $p_1 = p_2$. Check that the formula in part b gives the same result when $p_1 = p_2$.

HW 6.13 (2 points) The municipality of Randomburg provides free wifi for the citizens. Each wifi hotspot has $50m$ range, and they placed 3 hotsposts per square kilometer randomly. At the very moment when Bob left his house, the university sent an email to all the students to notify them that all teaching activity is cancelled for today due to a bomb alarm. Bob goes to university on a straight path, and he has 70% chance to receive the notification via the free wifi on the way. How far away is Bob's house from the university?