

Probability 1 – Exercises

Tutorial no. 2

14th Sep 2023

2.1 Let A , B and C be three events. Use set theory operations to express the following events:

- (a) Out of A , B and C , at least k events occurs ($k = 1, 2$).
- (b) Out of A , B and C , exactly k events occurs ($k = 0, 1, 2$).

2.2 How many different outcomes can the experiment have? Identify the sample space of the experiment:

- (a) We throw 3 different coins and 2 identical dice.
- (b) We throw 3 identical black dice, and 2 identical white dice.

2.3 What has a higher probability? Throwing a die four times, and rolling 6 at least once, or throwing two dice 24 times, and rolling 6 on both together at least once.

2.4 From a population of n balls in a bucket, we color R to be red, and the rest of them to be blue. Now, let's choose a sample of size N from the population. What is the probability that none of the selected balls are red if

- (a) we put them back into the bucket after each draw? (drawing with replacement)
- (b) we don't put them back into the bucket after each draw? (drawing without replacement)
- (c) Still drawing without replacement. What is the probability of choosing exactly i red balls, for $0 \leq i \leq \min\{N, R\}$ fixed?

2.5 There are 37 pockets on the roulette wheel: 18 black, 18 red and 1 green. We spin the wheel 13 times. What is the probability of rolling each color at least once?

HW 2.6 (2 points) We use a 52 card French card deck to play bridge. There are 4 players (N,E,S,W) and each get exactly 13 cards. What is the probability that N and S together have exactly k aces? ($k = 0,1,2,3,4$)

♣ 2.7 (3 points) Show that for any A , B events:

$$-\frac{1}{4} \leq \mathbb{P}(A)\mathbb{P}(B) - \mathbb{P}(A \cap B) \leq \frac{1}{4}$$

2.8 (a) Let A and B be two events. If $\mathbb{P}(A) \geq 0.8$ and $\mathbb{P}(B) \geq 0.5$ then prove the following:

$$\mathbb{P}(A \cap B) \geq 0.3$$

(b) Let's prove the following equality for any A_1, A_2, \dots, A_n events:

$$\mathbb{P}(A_1 \cap A_2 \cap \dots \cap A_n) \geq \mathbb{P}(A_1) + \mathbb{P}(A_2) + \dots + \mathbb{P}(A_n) - (n - 1)$$

HW 2.9 (3 points) (a) We throw a die 6 times. What is the probability, that 1, 2, 3, 4, 5 and 6 all get thrown?

(b) We throw a dice 10 times. What is the probability that 1, 2, 3, 4, 5 and 6 all get thrown at least once?

- 2.10** An urn contains 6 red, 4 blue, and 5 green balls. What is the sample space, if a set of 3 balls is randomly selected without replacement?
- (a) What is the probability that each of the balls will be of the same color?
- (b) What is the probability that each of the balls will be of different colors?
- HW 2.11** (2 points) There are 6 red, 6 white, and 7 blue balls in an urn. We draw 5 without replacement. What is the probability that we draw at least one of each color?
- 2.12** We are watching a horse race with 7 horses. Let A be the event that $HorseA$ is either first, second or third, while B is the event that $HorseB$ is second. What is the probability of $A \cup B$?
- 2.13** We deal all cards from a well shuffled French deck (52 cards). What is the probability that
- (a) the ace of hearts is the 11th distributed card?
- (b) the first distributed ace is the 11th distributed card?
- (c) the first 4 cards are of different suits? ($\clubsuit, \spadesuit, \heartsuit, \diamondsuit$)
- (d) an ace never follows the king of the same suit directly? (that is eg. $K \clubsuit A \clubsuit$ do not appear in the sequence)
- HW 2.14** (3 points) 10 couples sit down randomly to a round table. What is the probability that no husband sits next to his wife?
- 2.15** In a certain village, there are 20 families: 5 families with a single child, 7 families with 2, 4 families with 3, 3 families with 4 and 1 family with 5 children.
- (a) We choose a family at random. What is the probability that there are i children in this family? ($i = 1, \dots, 5$)
- (b) We choose a child at random. What is the probability that he has i siblings? ($i = 0, \dots, 4$)