Probability 1 – Excercises

Tutorial no. 1 7th Sep 2023

1.1 John, Jim, Jay and Jack have formed a band consisting of 4 instruments.

(a) If each boy can play all 4 instruments, how many different arrangements are possible?

(b) What if John and Jim can play all four instruments, but Jay and Jack can only play piano and drums?

1.2 For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9.

(a) How many area codes were possible?

(b) How many area codes starting with 4 were possible?

1.3 In how many ways can 8 people be seated in a row if

(a) there are no restrictions on the seating arrangement?

(b) persons A and B must sit next to each other?

(c) there are 4 men and 4 women, and no 2 men or 2 women can sit next to each other?

(d) there are 5 men and they must sit next to each other?

(e) there are 4 married couples, and each couple must sit together?

1.4 How many 5-card poker hands are there?

1.5 A dance class consists of 22 students, of which 10 are women, and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

HW 1.6 (1 point) A committee of 7, consisting of 2 Republicans, 2 Democrats, and 3 Independents, is to be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents. How many committees are possible?

1.7 How many different linear arrangements are there of the letters A,B,C,D,E,F,G for which

(a) A is before B?

(b) A is before B and B is before C?

(c) A and B are next to each other, and C and D are also next to each other?

(d) E is not last in line?

1.8 A bakery has 4 kinds of cakes. We want to order a tray of 20 pieces.

(a) How many different combinations for a tray are there?

(b) What if we insist on having at least one cake of each kind?

1.9 Consider a 3×5 grid (it means 15 junction points). Suppose that, starting at the bottom left corner $A = (1, 1)$, you can go one step up, or one step to the right at each move. This procedure is continued until the upper right corner $B = (5, 3)$ is reached. How many different paths from A to B are possible?

HW 1.10 (2 points) In the previous problem, how many different paths are there on a 5×8 grid from $A = (1, 1)$ to $B = (8, 5)$ that go through the point at $(3, 3)$?

HW 1.11 (2 points) (a) If 8 identical blackboards are to be divided among 2 schools, how many divisions are possible? How many if each school must receive at least 1 blackboard?

(b) What if 8 teachers (definitely not identical) are divided between two schools?

1.12 We put n (distinguishable) balls randomly into n (distinguishable) urns. What is the probability, that exactly one urn remains empty?

1.13 We put 8 rooks (tower) randomly onto a chess board. What is the probability that none of the rooks can knock each other out?

HW 1.14 (2 points) We have 10 mint and 10 lemon flavoured candies. The 20 candies are distributed randomly between Anne and Barbara, such that each girl gets 10 pieces. What is the probability that

(a) all the mint flavoured candies go to Anne?

(b) each girl gets exactly 5 pieces of each kind?

1.15 There are n pairs of shoes in a closet. We chose $2r$ shoes randomly ($2r \leq n$). What is the probability, that

(a) there are no pairs chosen?

(b) there is exactly one pair chosen?

(c) there are exactly two pairs chosen?

1.16 We throw a coin until it lands on the same side twice in a row. Prove that the probability of every n long sequence is 2^{-n}. Write down the probability space for the experiment. What is the probability of the following events?

$$A := \{\text{The experiment ends after less than 6 throws}\}$$
$$B := \{\text{The experiment ends after an even amount of throws }\}$$

HW 1.17 (3 points) Ariana, Britney and Cardi, are throwing coins, one-by-one. Ariana starts, then Britney, then Cardi, then Ariana again and so on. They continue, until someone throws heads. The one who throws heads first wins the game.

(a) What is the event space (Ω)?

(b) Write down the following events as subsets of the event space, and calculate their probabilities.

$$A = \{\text{Ariana wins}\}, \quad B = \{\text{Britney wins}\}, \quad (A \cup B)^c$$

1.18 Prove that:

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$

♣ **1.19** (3 points) Consider the following combinatorial identity:

$$\sum_{k=1}^{n} k \cdot \binom{n}{k} = n \cdot 2^{n-1}.$$

Present a combinatorial argument for this identity by considering a set of n people, and determining in two ways the number of possible selections of a committee of any size, and a chairperson for the committee.