Exam topics for Markov processes and martingales

2022/23 spring semester

1. Basic notions: σ-algebra, measurability, Lebesgue integral, monotone convergence, dominated convergence, Fatou lemma, Fubini theorem, random variables, independence, expectation, variance, covariance, Markov inequality, absolute continuity of measures, Radon–Nikodym theorem
2. Conditional expectation: definition, existence, uniqueness, construction for discrete partitions and with joint density, properties
3. Martingales: definition, examples, Pólya's urn, martingale transform
4. Stopping times, stopped martingales, Doob's optional stopping theorem, monkey at the typewriter problem
5. Definition of regular conditional distribution and its existence criterion, conditional characteristic function
6. Multivariate normal distribution: definition, affine transformations, bivariate case, independence and uncorrelatedness of marginals, conditioning normals
7. Hitting times for the simple random walk, superharmonic functions of Markov chains
8. Martingale convergence: Doob's upcrossing lemma, Doob's forward convergence theorem
9. Martingales bounded in L^2: Pythagorean formula, convergence in L^2
10. Sums of zero-mean independent random variables, Doob decomposition, angle bracket process
11. Convergence of an L^2 martingale and the finite limit of the angle bracket process
12. Cesàro's lemma, Kronecker's lemma, strong law under variance condition, strong law for L^2 martingales
13. Borel–Cantelli lemmas, Lévy's extension, closed martingales, closed L^p convergence, example of noisy observations
14. Reverse Fatou lemma, bounded convergence, absolute continuity of integration
15. Uniform integrability and L^1 convergence, uniform integrability and conditional expectation, uniformly integrable martingales, Lévy's upward theorem, Kolmogorov's $0-1$ law
16. Lévy's downward theorem, strong law of large numbers, Doob's submartingale inequality, Kolmogorov's inequality
17. Law of iterated logarithm
18. Doob's L^p inequality, Kakutani's theorem on product martingales
19. Stationary processes: examples, Kolmogorov's extension, measure preserving transformations, dynamical systems, ergodicity
20. Ergodic theorems: Neumann L^2 ergodic theorem, Birkhoff L^1 ergodic theorem, Weyl's equidistribution theorem
21. Central limit theorems: central limit theorem for martingales, central limit theorem for Markov chains
22. Markov chains and stopping times, reversible Markov chains, random walks on weighted graphs and electrical networks