

Exam topics for Markov processes and martingales

2020/21 spring semester

1. Basic notions: σ -algebra, measurability, Lebesgue integral, monotone convergence, dominated convergence, Fatou lemma, Fubini theorem, random variables, independence, expectation, variance, covariance, Markov inequality, absolute continuity of measures, Radon–Nikodym theorem
2. Conditional expectation: definition, existence, uniqueness, construction for discrete partitions and with joint density, properties
3. Martingales: definition, examples, Pólya’s urn, martingale transform
4. Stopping times, stopped martingales, Doob’s optional stopping theorem, monkey at the typewriter problem
5. Definition of regular conditional distribution and its existence criterion
6. Multivariate normal distribution: definition, affine transformations, bivariate case, independence and uncorrelatedness of marginals, conditioning normals
7. Hitting times for the simple random walk, superharmonic functions of Markov chains
8. Martingale convergence: Doob’s upcrossing lemma, Doob’s forward convergence theorem
9. Martingales bounded in L^2 : Pythagorean formula, convergence in L^2
10. Sums of zero-mean independent random variables, Doob decomposition, angle bracket process
11. Convergence of an L^2 martingale and the finite limit of the angle bracket process
12. Cesàro’s lemma, Kronecker’s lemma, strong law under variance condition, strong law for L^2 martingales
13. Borel–Cantelli lemmas, Lévy’s extension, closed martingales, L^p convergence, example of noisy observations
14. Reverse Fatou lemma, bounded convergence, absolute continuity of integration
15. Uniform integrability and L^1 convergence, uniform integrability and conditional expectation, uniformly integrable martingales, Lévy’s upward theorem, Kolmogorov’s 0 – 1 law
16. Lévy’s downward theorem, strong law of large numbers, Doob’s submartingale inequality, Kolmogorov’s inequality
17. Law of iterated logarithm
18. Doob’s L^p inequality, Kakutani’s theorem on product martingales
19. Stationary processes: examples, Kolmogorov’s extension, measure preserving transformations, dynamical systems, ergodicity
20. Ergodic theorems: Neumann L^2 ergodic theorem, Birkhoff L^1 ergodic theorem, Weyl’s equidistribution theorem
21. Central limit theorems: central limit theorem for martingales, central limit theorem for Markov chains
22. Markov chains and stopping times, reversible Markov chains, random walks on weighted graphs and electric networks