Képletgyűjtemény Matematika EP1 vizsgára

$\sin^2 x + \cos^2 x = 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$
$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
$\sin^2 x = \frac{1-\cos 2x}{2}$, $\quad \cos^2 x = \frac{1+\cos 2x}{2}$
$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$
$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$
$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$
$\sin x \sin y = -\frac{1}{2}[\cos(x+y) - \cos(x-y)]$
$\sinh x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$, $\quad \cosh x = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$
$\cosh^2 x - \sinh^2 x = 1$
$\sinh 2x = 2 \sinh x \cosh x$
$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\cosh^2 x = \frac{\cosh 2x + 1}{2}$, $\quad \sinh^2 x = \frac{\cosh 2x - 1}{2}$

Deriváltak

$(\sinh x)' = \cosh x$
$(\cosh x)' = \sinh x$
$(\log_a x)' = \frac{1}{x \ln a}$
$(x^\alpha)' = \alpha x^{\alpha-1}$
$(\mathrm{e}^x)' = \mathrm{e}^x$
$(a^x)' = a^x \ln(a)$
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \frac{1}{\cos^2 x}$
$(\cot x)' = -\frac{1}{\sin^2 x}$
$(\ln x)' = \frac{1}{x}$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{ar\,sinh} x)' = \frac{1}{\sqrt{1+x^2}}$
$(\operatorname{ar\,cosh} x)' = \frac{1}{\sqrt{x^2-1}}$
$(\operatorname{ar\,tanh} x)' = \frac{1}{1-x^2}$
$(\operatorname{ar\,coth} x)' = \frac{1}{1-x^2}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\operatorname{arc\,cot} x)' = -\frac{1}{1+x^2}$

Differenciálási szabályok

$(cu)' = cu'$ $\quad$ (c konstans)
$(u+v)' = u' + v'$
$(uv)' = u'v + uv'$
$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
$\frac{\mathrm{d}}{\mathrm{d}x} f(g(x)) = \frac{\mathrm{d}f}{\mathrm{d}g} \frac{\mathrm{d}g}{\mathrm{d}x}$

Integrálási szabályok

$\int cf \,\mathrm{d}x = c \int f \,\mathrm{d}x$ $\quad$ (c konstans)
$\int (f+g) \,\mathrm{d}x = \int f \,\mathrm{d}x + \int g \,\mathrm{d}x$
$\int f(ax+b) \,\mathrm{d}x = \frac{1}{a} F(ax+b) + c$,
ahol F az f primitív függvénye
$\int f(g(x)) g'(x) \,\mathrm{d}x = F(g(x)) + c$,
ahol F az f primitív függvénye
$\int f^\alpha f' \,\mathrm{d}x = \frac{f^{\alpha+1}}{\alpha+1} + c$, ha $\alpha \neq -1$
$\int \frac{f'}{f} \,\mathrm{d}x = \ln|f| + c$
$\int uv' \,\mathrm{d}x = uv - \int u'v \,\mathrm{d}x$

Nevezetes helyettesítések

$R(\mathrm{e}^x)$	$\mathrm{e}^x = t$
$R(\sqrt{ax+b})$	$\sqrt{ax+b} = t$
$R\left(\frac{\sqrt{ax+b}}{\sqrt{cx+d}}\right)$	$\frac{\sqrt{ax+b}}{\sqrt{cx+d}} = t$
$R(\sin x, \cos x)$	$\sin x, \cos x, \tan x, \tan \frac{x}{2} = t$
$R(x, \sqrt{a^2-x^2})$	$x = a \sin t,\ x = a \cos t$
$R(x, \sqrt{a^2+x^2})$	$x = a \sinh t$
$R(x, \sqrt{x^2-a^2})$	$x = a \cosh t$

Integrálok

$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + c$ $\quad$ ($\alpha \neq -1$)
$\int \mathrm{e}^{ax} \,\mathrm{d}x = \frac{1}{a} \mathrm{e}^{ax} + c$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + c$
$\int \cos x \,\mathrm{d}x = \sin x + c$
$\int \sin x \,\mathrm{d}x = -\cos x + c$
$\int \frac{1}{\cos^2 x} \,\mathrm{d}x = \tan x + c$
$\int \frac{1}{\sin^2 x} \,\mathrm{d}x = -\cot x + c$
$\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + c$
$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + c$
$\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$
$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \operatorname{ar\,sinh} \frac{x}{a} + c$
$\int \frac{\mathrm{d}x}{\sqrt{x^2-a^2}} = \operatorname{ar\,cosh} \frac{x}{a} + c$
$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{a} \operatorname{ar\,tanh} \frac{x}{a} + c$, $\quad$ ha $\left|\frac{x}{a}\right| < 1$
$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{a} \operatorname{ar\,coth} \frac{x}{a} + c$, $\quad$ ha $\left|\frac{x}{a}\right| > 1$
$\int \tan x \,\mathrm{d}x = -\ln|\cos x| + c$
$\int \cot x \,\mathrm{d}x = \ln|\sin x| + c$

Integrálás alkalmazásai

Terület: $T = \int_a^b f(x)\,\mathrm{d}x = \int_{t_1}^{t_2} y(t) \frac{\mathrm{d}x(t)}{\mathrm{d}t} \mathrm{d}t = \frac{1}{2} \int_{\varphi_1}^{\varphi_2} r^2(\varphi) \mathrm{d}\varphi$

Síkgörbe ívhossza: $s = \int_a^b \sqrt{1 + (f'(x))^2} \,\mathrm{d}x = \int_{t_1}^{t_2} \sqrt{\left(\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y(t)}{\mathrm{d}t}\right)^2} \mathrm{d}t = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2(\varphi) + \left(\frac{\mathrm{d}r(\varphi)}{\mathrm{d}\varphi}\right)^2} \mathrm{d}\varphi$

Forgástest térfogata: $V = \pi \int_a^b f^2(x)\,\mathrm{d}x = \pi \int_{t_1}^{t_2} y^2(t) \frac{\mathrm{d}x(t)}{\mathrm{d}t} \mathrm{d}t$

Forgástest felszíne: $A = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \,\mathrm{d}x = 2\pi \int_{t_1}^{t_2} y(t) \sqrt{\left(\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y(t)}{\mathrm{d}t}\right)^2} \mathrm{d}t$

Síkidom súlypontjának koordinátái: $x_s = \frac{\int_a^b x f(x)\,\mathrm{d}x}{\int_a^b f(x)\,\mathrm{d}x}$, $y_s = \frac{\frac{1}{2}\int_a^b f^2(x)\,\mathrm{d}x}{\int_a^b f(x)\,\mathrm{d}x}$