Exercise-book
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Part-II
Discrete distributions
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Contents

1 Discrete random variables and distributions

EXCEL

The following file shows several ways to visualize a discrete distribution, study them. If you are a beginner either in Excel or probability theory, then you should first study and learn to construct the figures called "Vertical bars ", "Dots with connected lines" and "Connected lines only" (pages 3, 7 and 8 in the file).

Demonstration file: Methods of visualization of discrete distributions
ef-120-01-00

Curious readers, in order to look at the most important discrete distributions, may study the following file, as well.

Demonstration file: Most important discrete distributions
ef-120-10-00
Demonstration file: Most important discrete distributions ef-120-10-00

PROBLEMS

1. *Calculating probabilities by summation, using Excel*
   The distribution of a random variable X is given by:

x	1	2	3	4	5	6	7	8	9	10
p(x)	0.01	0.04	0.06	0.09	0.10	0.15	0.25	0.20	0.07	0.03

   Calculate the probability that the value of the random variable is 4 or 8.

   Solution
   Sol-02-01-08

2. *Continuation of the previous problem*
   Invent other events related to the random variable X, and calculate their probabilities by summation.

3. *Calculating probabilities from a distribution*
   The distribution of a random variable X is given by:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

   (a) Calculate the probability that $3 \leq X \leq 6$.
   (b) Calculate the probability that $1 \leq X \leq 4$.

(c) Calculate the probability that $3 \leq X \leq 6$ on condition that $1 \leq X \leq 4$.

(d) Calculate the probability that $1 \leq X \leq 4$ on condition that $3 \leq X \leq 6$.

Solution
Sol-02-01-01

4. *Calculating probabilities from a distribution*
The distribution of a random variable X is given by:

k	1	2	3	...	99	100
$p(k)$	1/5050	2/5050	3/5050	...	99/5050	100/5050

(a) Calculate the probability that $30 \leq X \leq 60$;

(b) Calculate the probability that $10 \leq X \leq 40$;

(c) Calculate the conditional probability $P(30 \leq X \leq 60 \mid 10 \leq X \leq 40)$;

(d) Calculate the conditional probability $P(10 \leq X \leq 40 \mid 30 \leq X \leq 60)$.

Solution
Sol-02-01-02

5. *Calculating probabilities from a distribution*
Study the following file and observe how the outcomes are arranged: on page 1, they are listed in a column. On page 2, they constitute a rectangular table. The values of the probabilities are taken just from the air. The event A, whose probability is calculated, means that $X + Y$ is greater than 6 and less than 9.

Solution
Sol-02-01-20

6. *Calculating probabilities related to lottery*
There are 90 tickets in a box: 1, 2, 3, ... , 89, 90. We choose 5 of them without replacement as in the so called "90 choose 5 lottery".

(a) What is the probability that the biggest is 55?

(b) What is the probability that the biggest is 56?

(c) Let X denote the biggest number of the chosen ones. Find the distribution of X.

7. *Calculating probabilities related to lottery*
There are 90 tickets in a box: 1, 2, 3, ... , 89, 90. We choose 5 of them without replacement.

(a) What is the probability that the second biggest is 55?

(b) What is the probability that the second biggest is 56?

(c) Let X denote the second biggest number of the chosen ones. Find the distribution of X.

Solution
Sol-02-01-03

8. *Calculating probabilities related to lottery*
There are 90 tickets in a box: 1, 2, 3, ... , 89, 90. We choose 5 of them without replacement.

    (a) What is the probability that the third biggest is 55?

    (b) What is the probability that the third biggest is 56?

    (c) Let X denote the third biggest number of the chosen ones. Find the distribution of X.

9. *Largest and smallest of two dice*
Toss 2 dice, and let X be the largest, and Y be the smallest of the two numbers. Find out (make a numerical table for)

    (a) the distribution of X,

    (b) the distribution of Y .

    Solution 1
    Sol-02-01-04

    Solution 2
    Sol-02-01-05

10. *Absolute value of the difference*
Toss 2 dice, and observe the absolute value of the difference between the two numbers on the dice. Determine the distribution of this random variable.

    Solution 1
    Sol-02-01-06

    Solution 2
    Sol-02-01-07

11. *Number of heads with 3 dice*
Tossing with 3 coins observe the number of heads. Find the distribution of this random variable

12. *Number of heads with 5 dice*
Tossing with 5 coins observe the number of heads. Find the distribution of this random variable.

13. *Number of sixes with 4 dice*
Tossing with 4 dice observe the numbers sixes. Find the distribution of this random variable.

14. *Number of draws*
There are 2 red and 5 blue balls in a box. We draw without replacement until the first red is drawn. Let X denote the number of draws. Determine the distribution of X.

15. *Five people with red hats*
Five people, call them $1, 2, 3, 4, 5$, independently of each other put a red hat on their heads with certain (possibly different) probabilities. Clearly, there are $2^5 = 32$ possible variations.

    (a) Construct an Excel file to list the 32 variations in 32 rows, and then figure out an Excel formula to find the probability of each variation.

    (b) Determine the distribution of the number of people with a red hat.

    Solution
    Sol-02-01-12

16. *Ten people with red hats*
Ten people, call them $1, 2, \ldots, 10$, around a circular table, independently of each other put a red hat on their heads with certain (possibly different) probabilities. Clearly, there are $2^{10} = 1024$ possible variations.

    (a) Construct an Excel file to list the 1024 variations in 1024 rows, and then figure out an Excel formula to find the probability of each variation.
    Solution
    Sol-02-01-13

    (b) Determine the distribution of the number of people with a red hat.
    Solution
    Sol-02-01-14

    (c) A maximal set consisting of adjacent people with red hats is called a red island. Let Y be the number of red islands. Determine the distribution of Y .
    Solution
    Sol-02-01-15

    (d) A maximal set consisting of at least two adjacent people with red hats is called a big red island. Let Z be the number of big red islands. Determine the distribution of Z .
    Solution
    Sol-02-01-16

2 Uniform distribution (discrete)

EXCEL

The following file shows how a discrete uniform distribution looks like.

Demonstration file: Discrete uniform distribution
eg-020-60-52

PROBLEMS

17. *Is it uniformly distributed?*
If X is uniformly distributed on the set $\{-5, -4, ...4, 5\}$, and $Y = X^2$, then is Y also uniformly distributed?

18. *Is it uniformly distributed?*
If X is uniformly distributed on the set $\{0, 1, ...9, 10\}$, and $Y = X^2$, then is Y also uniformly distributed?

19. *Make a correct conclusion*
What conclusion can you draw for A and B if it is true that X is uniformly distributed on the set $\{A, A + 1, ...B - 1, B\}$, and $Y = X^2$ is also uniformly distributed on some set?

3 Hyper-geometrical distribution

EXCEL

The following file shows how a hyper-geometrical distribution looks like.

Demonstration file: Hyper-geometrical distribution
eg-020-60-21

PROBLEMS

20. *Number of red*
There are 24 balls in a box. 15 of them are red, 9 of them are blue. We draw 6 of them without replacement. What is the probability that among the chosen balls the number of red is

    (a) exactly 2;
    (b) less than 2;
    (c) less than or equal to 2;
    (d) more than 2?

    Solution
    Sol-02-03-14

21. *Tickets numbered and colored*
There are 45 tickets in a box. The tickets are numbered from 1 to 45, and they are also colored: tickets with numbers 1 through 6 are are red, the other 39 of them are blue. We draw 6 tickets without replacement. What is the probability that

    (a) among these 6 tickets, the number of red is an odd number, that is 1 or 3 or 5?
    (b) on these 6 tickets, the smallest number is the 10 and the largest is the 33?

4 Binomial distribution

EXCEL

The following files show how a binomial distribution distribution looks like. The first file should be studied for $n = 1, 2, ., 20$, the second for $n = 20, 21, ., 100$.

Demonstration file: Binomial distribution, when n is small
eg-020-60-01

Demonstration file: Binomial distribution, when n is large
eg-020-60-02

PROBLEMS

22. *Drawing with or without replacement*
    There are 25 balls in a box. 10 of them are red, 15 of them are blue. We draw 7 of them

    (a) with replacement;

    (b) without replacement.

    What is the probability that among the chosen balls the number of read is less than 2?

23. *Tossing a die*
    Toss a die 60 times. What is the probability that the number of sixes is

    (a) greater than or equal to 12;

    (b) less than or equal to 8;

    (c) strictly between 8 and 12 ?

24. *Blue eyed girls*
    Assume that 3/4 of girls in a country have blue eyes. If you choose 20 girls at random in that country, then what is the probability that

    (a) exactly 15 of them have blue eyes;

    (b) exactly 16 of them have blue eyes;

    (c) exactly 17 of them have blue eyes;

    (d) more than 17 of them have blue eyes;

    (e) less than 17 of them have blue eyes;

    (f) the number of blue eyed girls is between 15 and 17 (equality permitted)?

25. *Teachers becoming sick*
    There are 70 teachers in our institute. Each teacher, independently of the others, may become sick during a day with probability 0.04. What is the probability that k of them become sick during a day? Make a table and a figure - using binomial distribution - so that k runs from 0 to 20.

    Solution
    Sol-02-04-19

26. *Tickets in a box*
    There are N tickets in a box, numbered from 1 to N. Assume that $N = A + B$. The first A tickets are red, the other B are blue, that is, the tickets 1, 2, . . . , A are red, and $A + 1$, $A + 2$, . . . , $A + B$ are blue. We draw n times with replacement. What is the probability that we draw a red ticket exactly k times? For what k values is this probability greater than 0? For which k value is the probability maximal, that is, where is the mode of the distribution?

27. *Students attending or not*
Assume that each of 20 students attend a lesson independently of each other so that each attends with a probability 0.8. What is the probability that:

    (a) all attend;
    (b) nobody attends;
    (c) exactly 15 attend;
    (d) more than 15 attend;
    (e) less than 15 attend?

28. *How many chairs?*
Assume that each of 400 students attend a lesson independently of each other so that each attends with a probability 0.6. If the number of attending students is less than or equal to the number of chairs, then all students will have a chair to sit on. If the number of attending students is greater than the number of chairs, then some students will not have a chair to sit on, which is clearly a problem. For a given number of chairs the event we may ask: how much is the probability that no problem arises. It is obvious that in order hat this probability be equal to 1, the number of chairs must be 400 (or greater). Now the question is: how many chairs are needed so that this probability be (approximately) equal to 0.99?

    Solution
    Sol-02-04-22

29. *Air plane tickets*
Assume that there are 200 seats on an airplane. Since passengers may miss the flight, in order to get some extra profit the air line sells 5 extra tickets. Now assume that each of the 205 passengers miss the flight independently of each other with a probability 0.05. If the number of missing passenger is less than or equal to 5, then the number of passengers being at the flight is greater than 200 so some of them will not have a chair seat on the plane, and thus a problem arises.

    (a) What is the probability that problem will arise?
    (b) Replace the probability 0.05 and the number of extra tickets by other values, and analyze how the probability that "problem arises" depends on the probability of missing the plane and on the number of extra seats.
    (c) Make an analysis how the probability that "problem arises" depends on the size of the plane.

    Solution
    Sol-02-04-23

30. *Chess players*
Assume that each of 15 members of a group of chess players attend the club independently of each other so that each attends with a probability 0.8. If the number of attending members is odd, then one of them will not have a partner, and will be bored. What is the probability the number of attending members will be even, and so nobody will be bored?

31. *Opinion survey*
From the respondents who participated in an opinion survey, 50 percents declared to favor a unicameral parliament, 40 percents declared to favor a bicameral parliament, and 10 percents did not answer. 400 respondents from the interviewed sample are randomly chosen (For simplicity, you may assume: with replacement). What is the probability that exactly 200 of them were in favor of a unicameral parliament?

32. *Drawing with replacement*
If there are 10 balls in a box so that 8 are red and 2 are blue, and you draw 20 times with replacement, than how much is the probability that

    (a) you draw 14 times a red;
    (b) you draw less than 14 times a red;
    (c) you draw more than 14 times a red;
    (d) the number of red you draw is more than 12 and less than 18 times a red?

33. *Drawing with replacement*
If there are 40 balls in a box so that 8 are red and 32 are blue, and you draw 20 times with replacement, than how much is the probability that

    (a) you draw 6 times a red;
    (b) you draw less than 6 times a red,
    (c) you draw more than 6 times a red;
    (d) the number of red you draw is more than 2 and less than 8 times a red?

34. *Computer system braking down*
Assume that at a university, on each day, independently of other days, the central computer system brakes down with a probability 0.01.

    (a) What is the probability that during a year (365 days) the Central Computer System system never brakes down?
    (b) What is the probability that during a year there are more than 2 days when the Central Computer System system brakes down?

35. *Computers infected by viruses*
There are 50 computers in an office. Each of them, independently of the others, may become infected by viruses during a day with probability 0.05. What is the probability that k of them become infected by viruses during a day? Make a table and a figure - using binomial distribution - so that k runs from 0 to 20.

36. *Computers infected by viruses but still working*
There are 12 computers in an office. Each of them, independently of the others, has a virus with a probability 0.6. Each computer which has a virus still works, independently of the others, with a probability 0.7. The number of computers having a virus, but still working is a random variable, which we denote by W. Calculate the following probabilities:

    (a) $P(W = 5)$;
    (b) $P(W = k)$ for $j = 0, 1, ..., 12$.
    (c) Be convinced that the distribution you get is a binomial distribution with parameters 12 , and 0.42 . (Use Excel to make these calculations.)
    (d) Simulate W.

37. *Which box was used?*
There are two boxes: a red and a blue. In the red box there are 3 balls: 1 black and 2 white. In the blue box there are 5 balls: 3 black and 2 white. One of the boxes is chosen at random so that

    (a) each has the 50 percent chance;
    (b) the red box is 5 times probable than the blue.

    Then a ball is picked 20 times (with replacement) from that box. Assume that you get the information: how many times a black ball was picked, but you do not know which box was used. How would you guess the color of the box from the given information?

    Solution
    Sol-02-04-31

5 Geometrical distribution (pessimistic)

EXCEL

The following file shows how a pessimistic geometrical distribution looks like.

Demonstration file: Pessimistic geometrical distribution
eg-020-60-12

PROBLEMS

38. *Finding the first blue eyed girl*
Assume that 3/4 of girls in a country have blue eyes. If you choose girls at random until the first blue eyed one, then what is the probability that

(a) there are 2 not blue eyed girls before the first blue eyed girl;

(b) there are 3 not blue eyed girls before the first blue eyed girl;

(c) there are 2 or 3 not blue eyed girls before the first blue eyed girl.

(d) Let X be equal to the number of not blue eyed girls before the first blue eyed girl. Find the distribution of X.

39. *Asking for help on a highway*
When your car breaks down on a highway and you ask ask for help. Assume that each driver, independently of the other stops and helps you with a probability 0.2. What is the probability that

(a) exactly,

(b) at most,

(c) at least

5 cars pass without giving you help before somebody will help you?

6 Geometrical distribution (optimistic)

EXCEL

The following file shows how an optimistic geometrical distribution looks like.

Demonstration file: Optimistic geometrical distribution
eg-020-60-11

PROBLEMS

40. *Finding the firs blue eyed girl*
Assume that 3/4 of girls in a country have blue eyes. If you choose girls at random until the first blue eyed one, then what is the probability that

(a) the first blue eyed girl will be the 3rd girl;

(b) the first blue eyed girl will be the 4th girl?

(c) the first blue eyed girl will be the 3rd or the 4th girl.

(d) Let X be equal to the number of choices to find the first first blue eyed girl. Find the distribution of X.

41. *Rainy days in Budapest and in New York*
I live in Budapest, my friend in New York. We both calculate the number of days until the first rainy day. The probability of rain in Budapest is 0.3, in New York 0.4, each day, independently of each other and other days. My observation yields X, the observation of my friend yields Y. Both X and Y are random variables with possible values $1, 2, 3, \ldots$. Calculate the following probabilities:

    (a) $P(X = 5)$;
    (b) $P(X < 5)$;
    (c) $P(Y = 4$;
    (d) $P(X < 15 \text{ and } X > 4)$;
    (e) $P(X = 5 \text{ and } Y = 4)$,
    (f) $P(X < 15 \text{ and } X > 4sY < 4)$,
    (g) $P(Y = 2X)$;
    (h) $P(X < Y \text{ and } Y < 15)$;
    (i) $P(X < Y)$;
    (j) Simulate $X, Y)$.

42. *Apple trees blooming*
Let us assume that when the apple trees are blooming in Spring, the number of flowers on a tree follows geometrical distribution with an average of 50. Each flower, independently of the others, by the end of the summer will turn into an apple with a probability 2/3. Assuming that there are 30 apples on a tree at the end of the summer, what is the probability that there were 40 flowers on it in Spring? (You may leave the answer in the form of a sum.)

43. *Tossing a coin until the first head*
A coin is tossed as many times as needed to get the first head.

    (a) What is the probability that we have to make only 1 toss ?
    (b) What is the probability that we have to make 2 tosses ?
    (c) What is the probability that we have to make 3 tosses ?
    (d) What is the probability that we have to make 4 tosses ?
    (e) What is the probability that we have to make k tosses ? (Give a formula)

44. *Tossing a die until the first six*
A die is tossed as many times as needed to get the first six. What is the probability that we have to make only 1 toss? What is the probability that we have to make 10 tosses? What is the probability that we have to make k tosses (k is a positive integer)?

7 *** Negative binomial distribution (pessimistic)

EXCEL

The following file shows how a pessimistic negative binomial distribution looks like.

Demonstration file: Pessimistic negative binomial distribution
eg-020-60-32

PROBLEMS

45. *Tossing a coin until the second head*
A coin is tossed as many times as needed to get the 2nd head.

    (a) What is the probability that the number of tails before the 2nd head is 0?
    (b) What is the probability that the number of tails before the 2nd head is 1?
    (c) What is the probability that the number of tails before the 2nd head is 2?
    (d) What is the probability that the number of tails before the 2nd head is k? (Give a formula)

    What is the probability that the number of not-aces before the 2nd ace is 0?

46. *Tossing a coin until the second ace*
A die is tossed as many times as needed to get the 2nd ace. (Ace generally means the largest possible value, which is the six, here.)

    (a) What is the probability that the number of not-aces before the 2nd ace is 0?
    (b) What is the probability that the number of not-aces before the 2nd ace is 1?
    (c) What is the probability that the number of not-aces before the 2nd ace is 2?
    (d) What is the probability that the number of not-aces before the 2nd ace is k? (Give a formula)

47. *Tossing a coin until the third head*
A coin is tossed as many times as needed to get the 3rd head.

    (a) What is the probability that the number of tails before the rd head is 0?
    (b) What is the probability that the number of tails before the rd head is 1?
    (c) What is the probability that the number of tails before the rd head is 7?
    (d) What is the probability that the number of tails before the rd head is k? (Give a formula)

48. *Tossing a die until the third ace*
A die is tossed as many times as needed to get the 3rd ace.

(a) What is the probability that the number of not-aces before the 3rd ace is 0?

(b) What is the probability that the number of not-aces before the 3rd ace is 1?

(c) What is the probability that the number of not-aces before the 3rd ace is k? (Give a formula)

49. *Tossing a coin until the rth head*
A coin is tossed as many times as needed to get the rth head.

(a) What is the probability that the number of tails before the rd head is 0?

(b) What is the probability that the number of tails before the rd head is 1?

(c) What is the probability that the number of tails before the rd head is 2?

(d) What is the probability that the number of tails before the rd head is k? (Give a formula)

50. *Tossing a die until the rth ace*
A die is tossed as many times as needed to get the rth ace.

(a) What is the probability that the number of not-aces before the 3rd ace is 0?

(b) What is the probability that the number of not-aces before the 3rd ace is 1?

(c) What is the probability that the number of not-aces before the 3rd ace is 2?

(d) What is the probability that the number of not-aces before the 3rd ace is k? (Give a formula)

8 *** Negative binomial distribution (optimistic)

EXCEL

The following file shows how a pessimistic negative binomial distribution looks like.

Demonstration file: Optimistic negative binomial distribution
eg-020-60-31

PROBLEMS

51. *Tossing a coin until the second head*
A coin is tossed as many times as needed to get the 2nd head.

(a) What is the probability that we have to make 2 tosses?

(b) What is the probability that we have to make 3 tosses?

(c) What is the probability that we have to make 4 tosses?

(d) What is the probability that we have to make k tosses? (Give a formula)

52. *Tossing a die until the second ace*
A die is tossed as many times as needed to get the 2nd ace.

(a) What is the probability that we have to make 2 tosses?

(b) What is the probability that we have to make 3 tosses?

(c) What is the probability that we have to make 4 tosses?

(d) What is the probability that we have to make k tosses? (Give a formula)

53. *Tossing a coin until the third head*
A coin is tossed as many times as needed to get the 3rd head.

(a) What is the probability that we have to make 3 tosses?

(b) What is the probability that we have to make 4 tosses?

(c) What is the probability that we have to make 10 tosses?

(d) What is the probability that we have to make k tosses? (Give a formula)

54. *Tossing a die until the third ace*
A die is tossed as many times as needed to get the 3rd ace.

(a) What is the probability that we have to make 3 tosses?

(b) What is the probability that we have to make 4 tosses?

(c) What is the probability that we have to make k tosses? (Give a formula)

55. *Tossing a coin until the rth head*
A coin is tossed as many times as needed to get the rth head.

(a) What is the probability that we have to make r tosses?

(b) What is the probability that we have to make $r + 1$ tosses?

(c) What is the probability that we have to make $r + 2$ tosses?

(d) What is the probability that we have to make k tosses? (Give a formula)

56. *Tossing a die until the rth ace*
A die is tossed as many times as needed to get the rth ace.

(a) What is the probability that we have to make r tosses?

(b) What is the probability that we have to make $r + 1$ tosses?

(c) What is the probability that we have to make $r + 2$ tosses?

(d) What is the probability that we have to make k tosses? (Give a formula)

57. *Colored tickets in a box*
There are N tickets in a box, numbered from 1 to N. Assume that $N = A + B$. The first A tickets are red, the other B are blue, that is, the tickets 1, 2, . . . , A are red, and $A+1$, $A+2$, . . . , $A+B$ are blue. We draw with replacement until the rth time we get a red ticket.

(a) What is the probability that we draw exactly k times?

(b) For what k values is this probability different from 0?

(c) For which k value is the probability maximal, that is, where is the mode of the distribution?

9 Poisson-distribution

EXCEL

The following file shows how a Poisson-distribution looks like.

Demonstration file: Poisson-distribution
eg-020-60-41

PROBLEMS

58. *Computers infected by viruses*
There are 50 computers in an office. Each of them, independently of the others, may become infected by viruses during a day with probability 0.05.

(a) What is the probability that k of them become infected by viruses during a day?

(b) Make a table and a figure - using binomial distribution - so that k runs from 0 to 20.

(c) Make a table and a figure - using Poisson-distribution, too. Compare them.

59. *Teachers becoming sick*
There are 70 teachers in our institute. Each teacher, independently of the others, may become sick during a day with probability 0.04.

(a) What is the probability that k of them become sick during a day?

(b) Make a table and a figure - using binomial distribution - so that k runs from 0 to 20.

(c) Make a table and a figure - using Poisson-distribution, too. Compare them.

60. *Earthquakes*
Assume that in a certain country the average number of earthquakes during a year is 1.6. What is the probability that during a year the number of earthquakes is at most 2? What is the probability that during 3 years the number of earthquakes is at most 2? How many earthquakes are the most probable during a year, and during 3 years?

61. *Earthquakes in another country*
Assume that in an other country the probability that during a year at least one earthquake happens is $0,3$. What is the probability that during 5 years the number of earthquakes is at least 3?

62. *Shooting stars*
Assume that from a certain hill top during an August night one can see shooting stars so that the average amount of time between them is 10 minutes. What is the probability that during 15 minutes we see exactly 2 shooting stars?

63. *Accidents*
If the average number of accidents in a city during a day is 7.2, then what is the probability that the number of accidents in that city is

(a) less than 5;

(b) more than 10;

(c) is more 5 but less than 10?

(d) How many accidents are the most likely?

64. *Accidents in Buda and in Pest*
Assume that the average number of serious accidents in Buda during a day is 1.6, while in Pest it is $2,5$. What is the probability that during a day there is at least 1 serious accident in Pest, but no serious accident happens in Buda? How many serious accidents are the most probable in the whole territory of Budapest?

65. *Magnetic tapes*
Suppose that a certain type of magnetic tape contains, on the average, 3 defects per 1000 feet. What is the probability that a roll of tape 1200 feet long contains no defects?

66. *Twins*
Assume that the average number of twin births in a city during a day is 0.7. What is the probability that the number of twin births in that city during 3 days is 2? What is the most probable number of twins birth in that city during 3 days?

67. *Triple twins*
Assume that the average number of triple twins during a year in a country is 1.5. What is the probability that there are

(a) no triple twins;

(b) more than 2 triple twins during two and a half years? (Explain why the distribution you use is justified.)

68. *How many fish?*
Some years ago I met an old fisherman. He was fishing in a big lake, in which many small fish were swimming regardless of each other. He raised his from time to time, and picked the fish if there were any. He told me that out of 100 cases the net is empty only 15 times or so, and then he added: "If you could guess the number of fish in the net when I raise the net out of the water the next time, I would give you a big sack of money." I am sure he would not have said such a promise if he knew that his visitor was a well educated person in probability theory! I was thinking a little bit, then I made some calculation, and then a said a number. Which number did I say?

10 Higher dimensional discrete random variables and distributions

The following files shows some two-dimensional distributions.

Demonstration file:
Lottery, when 3 numbers are drawn out of 10
X_1 *= smallest lottery number*
X_2 *= largest lottery number*
Distribution of (X_1, X_2)
ef-120-10-55

Demonstration file:
90-lottery
X_1 *= smallest lottery number*
X_2 *= largest lottery number*
Distribution of (X_1, X_2)
ef-120-10-56

Demonstration file:
90-lottery
X_1 *= second smallest lottery number*
X_2 *= second largest lottery number*
Distribution of (X_1, X_2)
ef-120-10-57

Demonstration file:
Drawing from box1:

X_1 = when the first red occurs?
Drawing from box2:
X_2 = when the first blue occurs?
(X1 and X2 are independent)
Distribution of (X_1, X_2)
ef-120-10-59

Demonstration file:
Drawing from a box:
X_1 = when the first red occurs?
X_2 = when the first blue occurs?
(X_1 and X_2 are dependent)
Simulation for (X_1, X_2)
ef-120-10-60

Demonstration file:
Drawing from a box:
X_1 = when the first red occurs?
X_2 = when the first blue occurs?
(X_1 and X_2 are dependent)
Distribution of (X_1, X_2)
ef-120-10-61

PROBLEMS

69. *Drawing twice from a box without replacement*
    There are 6 tickets in a box numbered from 1 through 6. You choose two tickets, one after the other, without replacement. The first number is denoted by X, the second denoted is denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

70. *Drawing twice from a box without replacement*
    There are n tickets in a box numbered from 1 through n. You choose two tickets without replacement. The first number is denoted by X, the second is denoted denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

71. *Drawing twice from a box with replacement*
    There are 6 tickets in a box numbered from 1 through 6. You choose two tickets with replacement. The first number is denoted by X, the second is denoted denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

72. *Drawing twice from a box with replacement*
    There are n tickets in a box numbered from 1 through n. You choose two tickets with replacement. The first number is denoted by X, the second is denoted denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

73. *Drawing without replacement, when there are repeated tickets in the box*
    There are 6 tickets in a box with the numbers 1, 2, 2, 3, 3, 3. You choose two tickets without replacement. The first number is denoted by X, the second is denoted denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

74. *Drawing with replacement, when there are repeated tickets in the box*
    There are 6 tickets in a box with the numbers 1, 2, 2, 3, 3, 3. You choose two tickets with replacement. The first number is denoted by X, the second is denoted denoted by Y. Find the distribution of the two-dimensional random variable (X, Y).

75. *Calculating a conditional distribution*
    There are 6 tickets in a box numbered from 1 through 6. First you choose a ticket, and you denote the number on it by X. Then you remove from the box all the tickets which have a number greater than X. Then you choose a second number from the box, and you denote the number on it by Y. Obviously $Y \leq X$.

    (a) Find the distribution of the two-dimensional random variable (X, Y).
    (b) What is the probability that $Y = k$?
    (c) Assume that $Y = k$. Find the distribution of X under this condition.

76. *Calculating a conditional distribution*
    There are n tickets in a box numbered from 1 through n. First you choose a ticket, and you denote the number on it by X. Then you remove from the box all the tickets which have a number greater than X. Then you choose a second number from the box, and you denote the number on it by Y. Obviously $Y \leq X$.

    (a) Find the distribution of the two-dimensional random variable (X, Y).
    (b) What is the probability that $Y = k$?
    (c) Assume that $Y = k$. Find the distribution of X under this condition.

77. *Calculating a conditional distribution*
    There are 6 tickets in a box numbered from 1 through 6. First you choose a ticket, and you denote the number on it by X. Then you remove from the box all the tickets which have a number greater than X. Then you choose a second number from the box, and you denote the number on it by Y. Obviously $Y \leq X$.

    (a) What is the probability that $X + Y = 5$?
    (b) Assume that $Y = 5$. Find the distribution of $X + Y$ under this condition.

78. *Calculating a conditional distribution*
There are 6 tickets in a box numbered from 1 through 6. First you choose a ticket, and you denote the number on it by X. Then you remove from the box all the tickets which have a number greater than X. Then you choose a second number from the box, and you denote the number on it by Y. Obviously $Y \le X$.

(a) What is the probability that $X + Y \le 5$?

(b) Assume that $X + Y \le 5$. Find the distribution of $X + Y$ under this condition.

11 *** Poly-hyper-geometrical distribution

EXCEL

The following file shows a Poly-hyper-geometrical distribution.

Demonstration file: Poly-hyper-geometrical distribution
eg-020-01-00

PROBLEM

79. *Determining a poly-hyper-geometrical distribution*
There are 30 tickets in a box. 5 of them are red, 10 of them are white, 15 of them are green. You choose 7 tickets without replacement. X denotes how many times a red ticket is chosen, Y denotes how many times a white ticket is chosen. Recall that the distribution of X is the hyper-geometrical distribution with parameters (5;25;7), and the distribution of Y is the hyper-geometrical distribution with parameters (10;20;7). Find now the distribution of the two-dimensional random variable (X, Y).

12 *** Polynomial distribution

EXCEL

The following file shows a Polynomial distribution.

Demonstration file: Polynomial distribution
ef-020-02-00

PROBLEM

80. *Determining a polynomial distribution*
There are 30 tickets in a box. 5 of them are red, 10 of them are white, 15 of them are green. You choose 7 tickets with replacement. X denotes how many times a red ticket is chosen, Y denotes how many times a white ticket is chosen. Recall that the distribution of X is the binomial distribution with parameters $n = 7$, $p = 5/30$, and the distribution of Y is the binomial distribution with parameters $n = 7$, $p = 10/30$. Find now the distribution of the two-dimensional random variable (X, Y).

13 Generating a random variable with a given discrete distribution

EXCEL

The following file shows how a random variable with a given distribution can be simulated.

Demonstration file: Generating a random variable with a given discrete distribution
ef-130-00-00

PROBLEMS

81. *Simulating random variables with discrete uniform distribution*
Simulate with Excel
    (a) a random variable X uniformly distributed on the set $1, 2, 3, 4, 5, 6$;
    (b) a random variable X uniformly distributed on the set $11, 12, 13, 14, 15, 16$;
    (c) a random variable X uniformly distributed on the set $1, 4, 9, 16, 25, 36$;
    (d) a random variable X uniformly distributed on the set $1, 2, ...n$, where n is a parameter;
    (e) a random variable X uniformly distributed on the set $A, A+1, ...B$, where A and B are parameters.

82. *Simulating a random variable with non-uniform discrete distribution*
Simulate with Excel a random variable which has the following distribution:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

83. *Simulating a random variable with binomial distribution*
Simulate with Excel a random variable which has the binomial distribution with parameters

    (a) 10 and 0.5;
    (b) 10 and 0.7;
    (c) n and p, where n and p are parameters.

84. *Simulating a random variable with Poisson-distribution*
Simulate with Excel a random variable which has the Poisson-distribution with parameter

    (a) 2.8;
    (b) λ, where λ is a parameter.

85. *Simulating a random variable with pessimistic geometrical distribution*
Simulate with Excel a random variable which has the pessimistic geometrical distribution with parameter

    (a) 1/6;
    (b) p, where p is a parameter.

86. *Simulating a random variable with optimistic geometrical distribution*
Simulate with Excel a random variable which has the optimistic geometrical distribution with parameter

    (a) 1/6;
    (b) p, where p is a parameter.

87. *Simulating a random variable with pessimistic negative binomial distribution*
Simulate with Excel a random variable which has the pessimistic negative binomial distribution with parameters

    (a) 3 and 0.5;
    (b) 3 and p;
    (c) r and p, where r and p are parameters.

88. *Simulating a random variable with optimistic negative binomial distribution*
Simulate with Excel a random variable which has the optimistic negative binomial distribution with parameters

    (a) 3 and 0.5;
    (b) 3 and p;
    (c) r and p, where r and p are parameters.

14 Mode of a distribution

EXCEL

When a discrete distribution is given by a table in Excel, its mode can be easily identified. This is shown is the next file.

Demonstration file: Calculating - with Excel - the mode of a discrete distribution
ef-140-01-00

The next file shows the modes of some important distributions:

Demonstration file: Modes of binomial, Poisson and negative binomial distributions
ef-130-50-00

PROBLEMS

89. *Finding the mode*
    Find the mode(s) of the following distribution:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

90. *Finding the mode*
    Find the mode(s) of the following distribution:

k	1	2	3	4	5	6	7
$p(k)$	0.2	0.1	0.1	0.1	0.2	0.2	0.1

91. *Finding the mode of a binomial distribution*
    Find the mode(s) of the binomial distribution with parameters

    (a) 10 and 0.5;

    (b) 10 and 0.7;

    (c) n and p, where n and p are parameters.

92. *Finding the mode of a Poisson distribution*
    Find the mode(s) of the Poisson-distribution with parameter

    (a) 2.8;

    (b) λ, where λ is a parameter.

93. *Finding the mode of a pessimistic geometrical distribution*
Find the mode(s) of the pessimistic geometrical distribution with parameter

   (a) 1/6;
   (b) p, where p is a parameter.

94. *Finding the mode of an optimistic geometrical distribution*
Find the mode(s) of the optimistic geometrical distribution with parameter

   (a) 1/6;
   (b) p, where p is a parameter.

95. *Finding the mode of a pessimistic negative binomial distribution*
Find the mode(s) of the pessimistic negative binomial distribution with parameters

   (a) 3 and 0.5;
   (b) 3 and 0.7;
   (c) 3 and p;
   (d) r and p, where r and p are parameters.

96. *Finding the mode of an optimistic negative binomial distribution*
Find the mode(s) of the optimistic negative binomial distribution with parameters

   (a) 3 and 0.5;
   (b) 3 and 0.7;
   (c) 3 and p;
   (d) r and p, where r and p are parameters.

97. *Finding the mode of the birthday problem*
People chosen at random are asked which month and day they have their birthdays. We stop asking as soon as we get a birthday which has already been occurred before. Let X denote the number of people asked.

   (a) What is the distribution of X?
   (b) Find the mode of X?

15 Expected value of discrete distributions

EXCEL

The following file shows how the expected value of a discrete distribution can be calculated if the distribution is given by a table in Excel.

Demonstration file: Calculating the expected value of a discrete distribution ef-150-01-00

PROBLEMS

98. *Absolute value of the difference with two dice*
    Toss 2 dice, and observe the absolute value of the difference between the two numbers on the dice.
    (a) Calculate the expected value of this random variable.
    (b) Make 1000 simulations and be convinced that the average of the experimental results is close to the expected value.

99. *Number of heads with three coins*
    Tossing with 3 coins observe the number of heads. Find the expected value of this random variable

100. *Number of tosses*
    Toss a coin until you get the first time a head. How much is the expected value of the random variable X defined as the number of tosses.

101. *Number of sixes with four dice*
    Tossing with 4 dice observe the numbers sixes. Find the expected value of this random variable.

102. *Maximum with two dice*
    Tossing with 2 dice observe the maximum of the 2 numbers we toss. Find the expected value of this random variable.

103. *Number of draws*
    There are 2 red and 5 blue balls in a box. We draw without replacement until the first red is drawn. Let X denote the number of draws. Calculate the expected value of X.

104. *Number of tosses*
    Toss a pair of coins until you get the first time that both coins are heads. How much is the expected value of the random variable X defined as the number of tosses.

105. *Number of tosses*
    Toss a die until you get the first time an ace. How much is the expected value of the random variable X defined as the number of tosses.

106. *Number of tosses*
Toss a pair of dice until you get the first time that both dice are aces. How much is the expected value of the random variable X defined as the the number of tosses.

107. *Number of injured people*
Assume that when a 5 passenger car has an accident, then the number X of injured people, independently of any other factors, has the following distribution:

$P(X = 0) = 0.4,$
$P(X = 1) = 0.2,$
$P(X = 2) = 0.1,$
$P(X = 3) = 0.1,$
$P(X = 4) = 0.1,$
$P(X = 5) = 0.1,$

and when an 8 passenger bus has an accident, then the number Y of injured people, independently of any other factors, has the following distribution:

$P(Y = 0) = 0.50,$
$P(Y = 1) = 0.10,$
$P(Y = 2) = 0.10,$
$P(Y = 3) = 0.05,$
$P(Y = 4) = 0.05,$
$P(Y = 5) = 0.05,$
$P(Y = 6) = 0.05,$
$P(Y = 7) = 0.05,$
$P(Y = 8) = 0.05.$

(a) How much is the expected value of the number of injured people when a 5 passenger car has an accident?

(b) How much is the expected value of the number of injured people when an 8 passenger bus has an accident?

(c) How much is the expected value of the number of injured people when a 5 passenger car hits an 8 passenger bus?

108. *Mobile-phone calls during an hour*
Assume that the average number of mobile-phone calls a man gets during an hour is 2.5. What is the probability that he gets

(a) exactly 0;

(b) exactly 1;

(c) exactly 2;

(d) exactly 3;

(e) less than 2;

(f) more than 2

calls during an hour?

109. *Mobile-phone calls during two hours*
(Continuation of the previous problem.) Consider now the random variable: "the number of calls during 2 hours". Based on your common sense, figure out how much the its expected value is during 2 hours. What is the probability that he gets

(a) exactly 0;

(b) exactly 1;

(c) exactly 2;

(d) exactly 3;

(e) less than 2;

(f) more than 2

calls during 2 hours?

110. *Expected value of the number of draws*
There are four tickets in a box numbered from 1 to 4. We draw without replacement as many times as needed to get the ticket with the number 4.

(a) What is the probability that the number of draws is an even number?

(b) How much is the expected value of the number of draws?

111. *Comparison of the expected values*
A discrete distribution is defined by the formula: $p(x) = \frac{x^2}{30}$ $(x = 1, 2, 3, 4)$. Sketch the graph of the distribution. How much is its expected value? An other discrete distribution is defined by the formula: $p(x) = \frac{x^2}{7.5}$ $(x = \frac{1}{2}, 1, \frac{3}{2}, 2)$. Sketch the graph of the distribution. How much is its expected value? Compare them.

112. *Comparison of the expected values*
Calculate the numerical value of the expected value of the following distributions:

(a)

k	1	2	3	4	5	6	7	8	9	10
$p(k)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(b)

k	2	4	6	8	10	12	14	16	18	20
$p(k)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(c)

k	11	12	13	14	15	16	17	18	19	20
$p(k)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(d)

k	33	36	39	42	45	48	51	54	57	60
$p(k)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(e)

k	1	2	3	4
$p(k)$	0.4	0.3	0.2	0.1

(f)

k	1	2	3	4
$p(k)$	0.1	0.2	0.3	0.4

(g)

k	10	20	30	40
$p(k)$	0.1	0.2	0.3	0.4

(h)

k	-10	-20	-30	-40
$p(k)$	0.1	0.2	0.3	0.4

(i)

k	50	60	70	80
$p(k)$	0.1	0.2	0.3	0.4

Compare them.

16 Expected values of the most important discrete distributions

EXCEL

The following file gives the expected value of geometrical, binomial and Poisson-distributions.

Demonstration file: Expected values of the most important discrete distributions ef-150-07-01

PROBLEMS

113. *Lottery players*
Assume that during a year (52 weeks), the expected value of the number of lottery players winning 5 hits on the "5-lottery" is 3.4, the expected value of the number of lottery players winning 6 hits on the "6-lottery" is 7.6. What is the probability that

 (a) during a year, on the "5-lottery" nobody has 5 hits;
 (b) during a month (4 weeks), on the "5-lottery" nobody has 5 hits? (Assume that the "intensity" of lottery players is uniform, during the year.)
 (c) during a year, on the "5-lottery" 5 players have 5 hits and on the "6-lottery" 6 players have 6 hits;

(d) during a year, the number of players having 5 hits on the "5-lottery" plus the number of players having 6 hits on the "6-lottery" is exactly 7?

114. *Number of tosses*
Toss a coin until you get the first time a head. How much is the expected value of the random variable X defined as the number of tosses.

115. *Number of tosses*
Toss a die until you get the first time an ace. How much is the expected value of the random variable X defined as the number of tosses.

116. *Number of tosses*
Toss a pair of dice until you get the first time that both dice are aces. How much is the expected value of the random variable X defined as the number of tosses.

117. *Discovering a formula for an expected value*
Tossing a die until the first six, let X be the number of tosses. Make actually 10 experiments for this random variable, and - analyzing the experimental results - set up a simple relation between the average of the observed X-values and the relative frequency of six. Then imagining a large number of experiments, discover how the expected value of X can be expressed in terms of the probability of tossing a six.

118. *Discovering a formula for an expected value*
Tossing a die until the third six, let X be the number of tosses. Make actually or just imagine 10 experiments for this random variable, and - analyzing the experimental results - set up a simple relation between the average of the observed X-values and the relative frequency of six. Then imagining a large number of experiments, discover how the expected value of X can be expressed in terms of the probability of tossing a six.

119. *Discovering a formula for an expected value*
Finally imagine that we toss a false die until the rth six, (false means that the probability of six is not necessarily 1/6, but it is, say, equal to p) and let X be the number of tosses. Imagine a large number of experiments for this random variable, and figure out a simple relation between the average of the observed X-values and the relative frequency of six in order to conclude a formula for the expected value of X.

120. *Deriving the expected value of the optimistic negative binomial distribution*
Here is a method to find the expected value of the optimistic negative binomial distribution: Imagine that we toss a false die until the 3rd six.
Let $X =$ the number of tosses to get the 3rd six.
Introduce the following random variables, as well:
$X_1 =$ the number of tosses to get the 1st six.
$X_2 =$ the number of tosses after the 1st six to get the 2nd six.
$X_3 =$ the number of tosses after the 2nd six to get the 3rd six.
On one hand, X_1, X_2, X_3 obviously follow the optimistic geometrical distribution with parameter p, so their expected value is $\frac{1}{p}$. On the other hand, a simple

relation between X and X_1, X_2, X_3 can be noticed.. From this relation it is easy to derive the formula for the expected value of X.

17 Expected value of a function of a discrete random variable

EXCEL

The following file shows how the expected value of a function of a discrete random variable can be calculated if the distribution of the random variable is given by a table in Excel.

Demonstration file: Calculating - with Excel - the expected value of a function for a discrete distribution
ef-160-01-00

PROBLEMS

121. *Expected values of some functions of a discrete random variable*
The distribution of a random variable X is given by:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

How much is its expected value of

(a) X;

(b) $1/X$;

(c) X^2?

122. *Expected values of some functions of a discrete random variable*
Assume that the random variable X follows the discrete distribution: $p(x) = \frac{x^2}{30}$ $(x = 1, 2, 3, 4)$. How much is its expected value of

(a) X?

(b) $1/X$?

(c) X^2?

18 Moments of a discrete random variable

EXCEL

The following files show how the moments of a discrete distribution can be calculated if the distribution is given by a table in Excel.

Demonstration file: Calculating the second moment of a discrete distribution
ef-170-01-00

PROBLEMS

123. *Second moments of some functions of a discrete random variable*
The distribution of a random variable X is given by:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

How much is the second moment of

(a) X;
(b) $1/X$;
(c) X^2?

124. *Second moments of some functions of a discrete random variable*
We toss two fair dice. How much is the second moment of

(a) the difference ("red die minus blue die");
(b) the minimum?

125. *Second moments of some functions of a discrete random variable*
Assume that the random variable X follows the discrete distribution: $p(x) = \frac{x^2}{30}$ $(x = 1, 2, 3, 4)$. How much is the second moment of

(a) X;
(b) $1/X$;
(c) X^2?

126. *Third moments of some function of a discrete random variable*
The distribution of a random variable X is given by:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

How much is the third moment of

(a) X;

(b) $1/X$;

(c) X^2?

127. *Third moments of some function of a discrete random variable*
Assume that the random variable X follows the discrete distribution: $p(x) = \frac{x^2}{30}$ $(x = 1, 2, 3, 4)$. How much is the third moment of

(a) X;

(b) $1/X$;

(c) X^2?

19 Projections and conditional distributions for discrete distributions

EXCEL

Here are some files to study the relations between projections and conditional distributions for discrete distributions.

Demonstration file: Construction from conditional distributions, discrete case (version A)
ef-200-75-00

Demonstration file: Construction from conditional distributions, discrete case (version B)
ef-200-76-00

Demonstration file: Projections and conditional distributions, discrete case (version A)
ef-200-77-00

Demonstration file: Projections and conditional distributions, discrete case (version B)
ef-200-78-00

PROBLEMS

128. *Unconditional and conditional probabilities and distributions*
Assume that the distribution of the random variable (X,Y) is

5	0.001	0.002	0.003	0.004	0.005	0.005
4	0.001	0.002	0.003	0.004	0.005	0.005
3	0.001	0.002	0.003	0.004	0.005	0.005
2	0.001	0.002	0.003	0.004	0.005	0.005
1	0.001	0.002	0.003	0.004	0.005	0.005
y/x	1	2	3	4	5	6

(a) Check that the sum of all probabilities is equal to 1.
(b) Calculate the probability $P(X + Y = 6)$.
(c) Calculate the probability $P(X + Y >= 6)$.
(d) Calculate the probability $P(X - Y >= 3)$.
(e) Calculate the conditional probability $P(X - Y >= 3 | X + Y >= 6)$.
(f) Determine the distribution of X.
(g) Determine the distribution of Y.
(h) Determine the conditional distributions of Y on condition that $X = 3$.
(i) Determine the conditional distributions of Y on condition that $X = 4$.
(j) Determine the conditional distributions of Y on condition that $X = x$ for all x.
(k) Determine the conditional distributions of X on condition that $Y = y$ for all y.
(l) Calculate the conditional probability $P(2 <= Y <= 5 | X = x)$ for all x.
(m) Are X and Y independent of each other?

129. *Unconditional and conditional probabilities and distributions*
Assume that the distribution of the random variable (X,Y) is

5	0.003	0.001	0.005	0.005	0.001	0.002
4	0.001	0.004	0.005	0.005	0.001	0.007
3	0.005	0.004	0.000	0.005	0.004	0.000
2	0.003	0.007	0.000	0.010	0.000	0.000
1	0.000	0.007	0.015	0.000	0.000	0.000
y / x	1	2	3	4	5	6

(a) Check that the sum of all probabilities is equal to 1.
(b) Calculate the probability $P(X + Y = 6)$.
(c) Calculate the probability $P(X + Y >= 6)$.
(d) Calculate the probability $P(X - Y >= 3)$.
(e) Calculate the conditional probability $P(X - Y >= 3 | X + Y >= 15)$.
(f) Determine the distribution of X.

(g) Determine the distribution of Y.

(h) Determine the conditional distributions of Y on condition that $X = 3$.

(i) Determine the conditional distributions of Y on condition that $X = 4$.

(j) Determine the conditional distributions of Y on condition that $X = x$ for all x.

(k) Determine the conditional distributions of X on condition that $Y = y$ for all y.

(l) Calculate the conditional probability $P(2 <= Y <= 5|X = x)$ for all x.

(m) Are X and Y independent of each other?

20 Transformation of discrete distributions

EXCEL

The following files give simple numerical examples for a transformations of a discrete distributions.

Demonstration file: Transformation of a discrete distribution
eg-020-03-01

Demonstration file: Transformation of a discrete distribution
eg-020-03-02

PROBLEMS

130. *Transformations of a discrete distribution*
The distribution of a random variable X is given by:

k	1	2	3	4	5	6	7
$p(k)$	0.1	0.1	0.1	0.1	0.3	0.2	0.1

Find the distribution of

(a) $2X$;

(b) X^2;

(c) $|X - 3|$.

131. *Transformations of a discrete distribution*
Assume that the distribution of the random variable (X,Y) is

5	0.001	0.002	0.003	0.004	0.005	0.005
4	0.001	0.002	0.003	0.004	0.005	0.005
3	0.001	0.002	0.003	0.004	0.005	0.005
2	0.001	0.002	0.003	0.004	0.005	0.005
1	0.001	0.002	0.003	0.004	0.005	0.005
y/x	1	2	3	4	5	6

Find the distribution of

(a) $X + Y$;
(b) $X - Y$;
(c) $2X + Y$.

132. *Transformations of a discrete distribution*
Assume that the distribution of the random variable (X,Y) is

5	0.003	0.001	0.005	0.005	0.001	0.002
4	0.001	0.004	0.005	0.005	0.001	0.007
3	0.005	0.004	0.000	0.005	0.004	0.000
2	0.003	0.007	0.000	0.010	0.000	0.000
1	0.000	0.007	0.015	0.000	0.000	0.000
y / x	1	2	3	4	5	6

Find the distribution of

(a) $X + Y$;
(b) $X - Y$;
(c) $2X + Y$.