First Midterm Test (2016 10 26) – SOLUTIONS

1. Assume that, in a country, 0.01 part of dogs are infected by a certain illness, and 0.99 part of them are healthy. Assume also that if a dog is infected by the illness, then it will be correctly diagnosed sick with a probability 0.8, and it will be mistakenly diagnosed healthy with a probability 0.2. Moreover, if a dog is healthy, then it will be correctly diagnosed healthy with a probability 0.7. and it will be mistakenly diagnosed sick with a probability 0.3, Now imagine that a dog is examined, and the test says the dog is sick. Knowing this fact what is the probability that this dog is really sick?

   Solution:

$$\text{P(sick} \mid \text{diagnosed sick)} = \frac{\text{P(sick AND diagnosed sick)}}{\text{P(diagnosed sick)}} =$$

$$= \frac{\text{P(sick AND diagnosed sick)}}{\text{P(sick AND diagnosed sick)} + \text{P(healthy AND diagnosed sick)}} =$$

$$= \frac{\text{P(sick) P(diagnosed sick} \mid \text{sick)}}{\text{P(sick)} \cdot \text{P(diagnosed sick} \mid \text{sick)} + \text{P(healthy)} \cdot \text{P(diagnosed sick} \mid \text{healthy)}} =$$

$$= \frac{(0.01)\ (0.8)}{(0.01)\ (0.8) + (0.99)\ (0.3)} \qquad (= 0.026)$$

2. Assume that there are 50 seats in a restaurant, and 55 guests are invited for a dinner. Let us assume that each guest may miss the dinner independently of the others with a probability $p = 0.07$. How much is the probability that more than 50 people come to the dinner?

   Solution:

$$\text{P(more than 50 guests come)} = \sum_{k=51}^{55} \text{P(} k \text{ guests come)} = \sum_{k=51}^{55} \binom{55}{k} (0.93)^k (0.07)^{55-k} \qquad (= 0.66)$$

3. X has an exponential distribution with parameter $\frac{1}{4}$. Find the probabilities:

   (a) $\text{P}(X < 3.5)$;

   (b) $\text{P}(X < 3.5 \mid X > 2.5)$;

   Solution:

   (a)

$$\text{P}(X < 3.5) = F(3.5) = 1 - \text{e}^{(-\frac{1}{4} \cdot 3.5)} = 1 - \text{e}^{(-\frac{7}{8})} \qquad (= 0.58)$$

   (b)

$$\text{P}(X < 3.5 \mid X > 2.5) = \frac{\text{P}(X < 3.5 \text{ AND } X > 2.5)}{\text{P}(X > 2.5)} = \frac{\text{P}(2.5 < X < 3.5)}{\text{P}(X > 2.5)} =$$

$$= \frac{F(3.5) - F(2.5)}{1 - F(2.5)} = \frac{\left(1 - \text{e}^{(-\frac{1}{4} \cdot 3.5)}\right) - \left(1 - \text{e}^{(-\frac{1}{4} \cdot 2.5)}\right)}{1 - \left(1 - \text{e}^{(-\frac{1}{4} \cdot 2.5)}\right)} = \frac{\text{e}^{(-\frac{1}{4} \cdot 2.5)} - \text{e}^{(-\frac{1}{4} \cdot 3.5)}}{\text{e}^{(-\frac{1}{4} \cdot 2.5)}} \qquad (= 0.06)$$

4. Explain the meaning of the the memoryless property

   (a) by a mathematical formula

   (b) in words. *(You may tell a real life example.)*

   Solution:

   (a)

$$\text{P}\,(\ X > s + t \mid X > t\ ) = \text{P}\,(\ X > s\ ) \quad \text{for all} \quad s, t \geq 0$$

   (b) The fact that life-times of the glasses at a restaurant satisfy the memoryless property means that the future life-time of a glass does not depend on its age.