Tests: $\qquad$ Exam: $\qquad$ Total: $\qquad$ GRADE: $\qquad$

NAME: $\qquad$ NEPTUN: $\qquad$

## Exam 20170109 , 90 minutes

1. In a box there are 5 red and 3 blue balls. You pick balls, one after the other without replacement. You stop picking when two red has already been drawn. Let $X$ mean the number of draws.
(a) Set up a table for the distribution of $X$.
(b) If you made 1000 experiments, approximately how much would be the average of the number of draws?
2. The life-time of an object has a uniform distribution between 0 and $B$. Approximately only $25 \%$ of such objects live more than 6 years.
(a) How much is the expected value of these objects?
(b) What is the probability that such an object lives more than 6 years on condition that it lives more than 5 years?
3. The height of a randomly chosen man follows the normal distribution with an expected value of 180 cms and a standard deviation of 10 cms . The height of a randomly chosen woman follows the normal distribution with an expected value of 170 cms and a standard deviation of 5 cms . In a large group of people $25 \%$ are men, $75 \%$ are women. You choose persons one after the other until a the person is higher than 190 cms , and then you stop choosing.
(a) What is the probability that you make exactly 4 choices?
(b) What is the probability that the person is a woman on condition that you make exactly 4 choices?
4. $X$ is a random variable with values between $-\infty$ and 0 . The density function of $X$ is $f(x)=3 \mathrm{e}^{3 x}$ on the interval $-\infty ; 0$.
(a) What is the probability that $-2.5<X<-0.5$ ?
(b) Determine the expected value of $X$.
5. $(X, Y)$ follows the distribution which has the density function

$$
f(x, y)=\frac{2 x}{y} \quad\left(0<x<1, x<y<\frac{1}{x}\right)
$$

(a) Find the density function of $X$.
(b) Find the conditional expected value of $Y$ on condition that $X=x$.
6. Give the meaning of the variance of
(a) the data set $\{1 ; 3 ; 7 ; 8 ; 11\}$ by making simple calculations (without using calculator). (Show the details of your calculations.)
(b) a continuous random variable by a correct(!) mathematical formula.

Standard normal distribution function

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0,0 | 0,50 | 0,5 | 0,69 | 1,0 | 0,84 | 1,5 | 0,93 | 2,0 | 0,98 | 2,5 | 0,99 |
| 0,1 | 0,54 | 0,6 | 0,73 | 1,1 | 0,86 | 1,6 | 0,95 | 2,1 | 0,98 | 2,6 | 1,00 |
| 0,2 | 0,58 | 0,7 | 0,76 | 1,2 | 0,88 | 1,7 | 0,96 | 2,2 | 0,99 |  |  |
| 0,3 | 0,62 | 0,8 | 0,79 | 1,3 | 0,90 | 1,8 | 0,96 | 2,3 | 0,99 |  |  |
| 0,4 | 0,66 | 0,9 | 0,82 | 1,4 | 0,92 | 1,9 | 0,97 | 2,4 | 0,99 |  |  |

