SHORT PROOF

E. T. SCHMIDT

Theorem 1. *(Grätzer and Knapp* [1], [2]*) Each finite slim planar semimodular lattice can be obtained from a cover-preserving join-homomorphic image of two finite chains.*

Proof. It is a trivial remark that every finite lattice L is the join-homomorphic image of a disztributive lattice B, $\varphi(B) = L$: ineed the free kommutative, idempotent semilattices are the finite Boolean lattices.

If L is semimodular then φ is "semimodular-preseving". This is just the cover-preserving property,

We have now a distributive lattice B and a cover-preserving join-homomorphism such that $\varphi(B) = L$. A short trivial discussion shows that B is planar. □

References

[1] G. Grätzer, E. Knapp, *Notes on planar semimodular lattices. I. Constructions,* Acta Sci. Math. (Szeged) **73** (2007), 445–462.

[2] G. Grätzer, E. Knapp, *A note on semimodular planar lattices,* Algebra Universalis. **58** (2008), 497–499.

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