FANO LATTICES

In [1], G. Grätzer and R. W. Quackenbush determined the subdirect irreducible members of the variety generated by the planar modular lattices. These results can easily extended to higher dimensions.

Let D be a planar distributive lattice. The basic method in [1] is the adding new elements to some of the covering squares to get covering M_3-s.

Take the finite field $\mathbf{GF}(p^n)$, $p = 2, n = 1$. The corresponding two-dimensional projective geometry $\mathbf{GP_2}$ is the *Fano-plane*. The one-dimensional lattice $\mathbf{GP_1}$ is M_3.

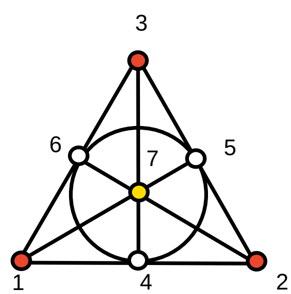

Figure 1. The Fano plane given by points and lines

On the next picture you can see the "traditional" presentation of the subspace lattice.

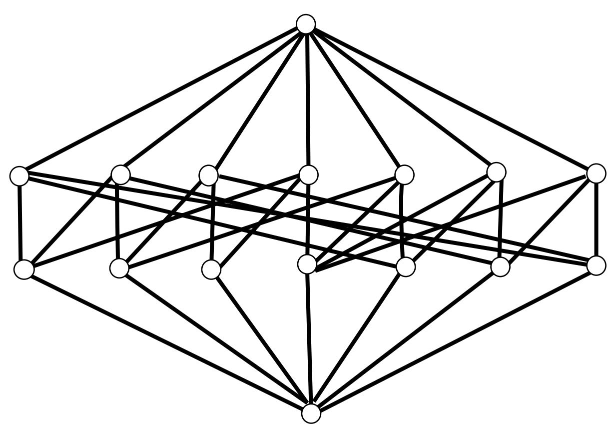

Figure 2. The "traditional" diagram of the subspace lattice of the Fano plane

Date: September 5, 2010.

Draw the diagram a little bit others we get the following diagram for the same lattice (the same, but differently).

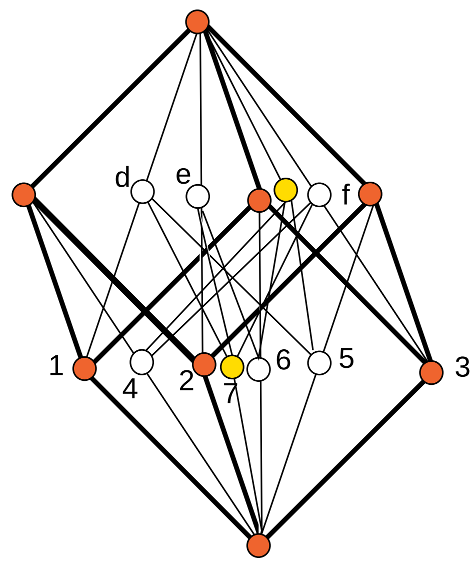

FIGURE 3. The subspace lattice of the Fano plane

Here we see a cube (fat lines, the skeleton) and six circles on the faces of this cube and two circles (yellow) are inside the cube. If D is the direct product of three chains then this contains unit (covering) cubes. We can extend D if we put the Fano-plane into some covering cubes. (Fano plane "locked" in a cube.) By plain lattices we extend a covering square to an M_3.

As an example let us see how does it works the attaching:

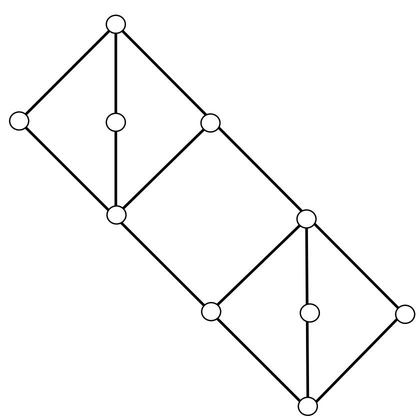

FIGURE 4. Attached lattice in [1]

The attaching of covering Fano plane in the three-dimensional case.

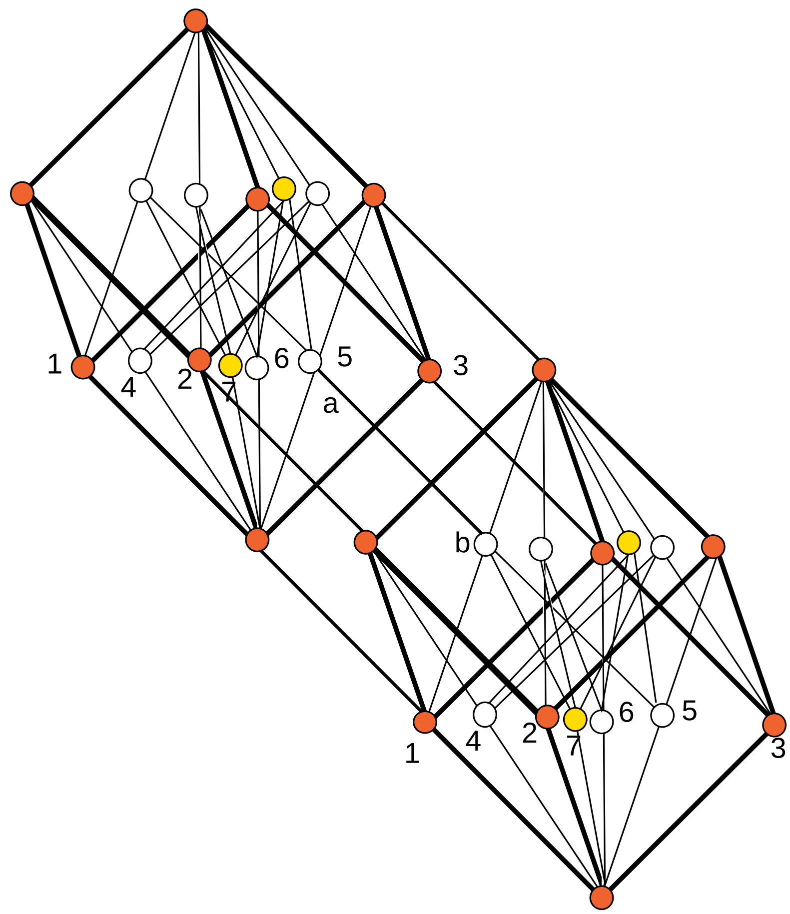

Figure 5. Attached Fano lattice

We define the Fano lattice perhaps as follows:

Definition. *Let D be the subdirect product of three chains. If we extend some of the covering cubes to Fano plane and some of the covering squares to M_3 the resulting lattice will be called a* Fano lattice (let be careful: as on Figure 5. we have a new edge $\overline{ab}$).

Every modular planar lattice which does not contains M_4 is a Fano lattice.

Problem. *Determine the subdirect irreducible members of the variety generated by the Fano lattices.*

References

[1] G. Grätzer, R. W. Quackenbush, *The variety generated by the planar modular lattices* Algebra Universalis **63** (2010), xxx–yyy.