

Midterm Exam - March 13, 2018, Limit thms. of probab.

Family name _____ Given name _____

Signature _____ Neptun Code _____

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (8 marks) Let $S_n = X_1 + \dots + X_n$, where X_1, X_2, \dots are i.i.d. with POI(1) distribution. Give a good upper bound on the probability $\mathbb{P}(S_n \geq e \cdot n)$. Write down all the details: calculate the relevant moment generating function, give a bound on $\mathbb{P}(S_n \geq e \cdot n)$ using the exponential Chebyshev's inequality, optimize your bound over the parameter λ of the moment generating function, simplify your result as much as possible.
2. (7 marks) Let $f(x) = \frac{1}{4}x^{-5/4}\mathbf{1}[x \geq 1]$. Let Y_1, Y_2, \dots denote i.i.d. random variables with probability density function $f(x)$. Denote by $M_n = \max\{Y_1, \dots, Y_n\}$. Find the value of $\beta \in \mathbb{R}_+$ for which M_n/n^β converges in distribution to a non-degenerate probability distribution as $n \rightarrow \infty$ and identify the c.d.f. of the limiting distribution.