

Answers to Selected Exercises

0. Foundations

1. Sets
2. Functions
3. Relations
4. Partial Orders
5. Equivalence Relations
7. Counting Measure
8. Combinatorial Structures

1. Sets

☑ 1.28

- a. $\{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)\}$
- b. $\{(1, 6), (2, 5), (3, 4), (4, 3)\}$
- c. $\{(2, 5)\}$
- d. $\{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (1, 6), (3, 4), (4, 3)\}$
- e. $\{(2, 1), (2, 2), (2, 3), (2, 4), (2, 6)\}$
- f. $\{(1, 6), (3, 4), (4, 3)\}$

☑ 1.29

- a. $\{000, 100, 010, 001, 110, 101, 011, 111\}$
- b. $\{010, 110, 011, 111\}$
- c. $\{110, 011, 101\}$
- d. $\{000, 100, 001, 101\}$
- e. $\{000, 100, 010, 001, 111\}$
- f. $\{110, 011\}$
- g. $\{010, 110, 011, 111, 101\}$
- h. $\{010, 111, 101\}$
- i. $\{101\}$

☑ 1.30

$\{\emptyset, \{00\}, \{01\}, \{10\}, \{11\}, \{00, 01\}, \{00, 10\}, \{00, 11\}, \{01, 10\}, \{01, 11\}, \{10, 11\}, \{00, 01, 10\}, \{00, 01, 11\}, \{00, 10, 11\}, \{01, 10, 11\}, \{00, 01, 10, 11\}\}$

☑ 1.31

- a. $\{j♥, q♥, k♥\}$
- b. $\{1♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥\}$
- c. $\{j♠, q♠, k♠, j♦, q♦, k♦, j♣, q♣, k♣\}$
- d. $\{1♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥, j♠, q♠, k♠, j♦, q♦, k♦, j♣, q♣, k♣\}$

☑ 1.32

- a. $\{0\}$
- b. $[0, 1)$
- c. $(-\infty, 0) \cup [1, \infty)$
- d. $\mathbb{R} \setminus \{0\}$

☑ 1.33

- a. $[2, 5]$
- b. $(1, 6)$
- c. $(-\infty, 1] \cup [6, \infty)$
- d. $(-\infty, 2) \cup (5, \infty)$

2. Functions

☑ 2.15

- a. Range $[0, \infty)$. Not one-to-one.
- b. Range $[-1, 1]$. Not one-to-one.
- c. Range \mathbb{Z} . Not one-to-one.
- d. Range $(0, 1)$. One-to-one.

☑ 2.16

- a. $[-3, -2] \cup [2, 3]$
- b. $\{n\pi : n \in \mathbb{Z}\}$
- c. $(1, 5]$

☑ 2.17 $F^{-1}(p) = \ln\left(\frac{p}{1-p}\right)$ for $p \in (0, 1)$

☑ 2.18

- a. $(f \circ g)(x) = \sin(x)^2$. Range $[0, 1]$
- b. $(g \circ f)(x) = \sin(x^2)$. Range $[-1, 1]$
- c. $(h \circ g \circ f)(x) = \lfloor \sin(x^2) \rfloor$. Range $\{-1, 0, 1\}$

☑ 2.19

- a. $\{2, 3, 4, \dots, 12\}$
- b. $\{1, 2, 3, 4, 5, 6\}$
- c. $\{1, 2, 3, 4, 5, 6\}$
- d. $\{(i, j) \in \{1, 2, 3, 4, 5, 6\}^2 : i \leq j\}$

☑ 2.20

- a. $\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$
- b. $\{(3, 3), (3, 4), (4, 3), (3, 5), (5, 3), (3, 6), (6, 3)\}$
- c. $\{(1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)\}$
- d. $\{(3, 4), (4, 3)\}$

3. Relations

☑ 3.9

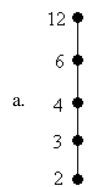
- a. yes
- b. yes
- c. yes
- d. no
- e. no

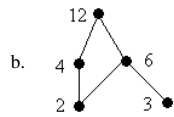
☑ 3.9

- a. no
- b. yes
- c. no
- d. no
- e. no

4. Partial Orders

☑ 4.6





4.13

- a. $\{a\}$
- b. \emptyset
- c. a
- d. Does not exist
- e. $[-\infty, a]$
- f. $[b, \infty]$
- g. a
- h. b

4.15

- a. $\{2, 3\}$
- b. $\{12\}$
- c. Does not exist
- d. 12
- e. $\{1\}$
- f. $\{12, 24, 36, \dots\}$
- g. 1
- h. 12

5. Equivalence Relations

5.7.

- a. $[x] = \{x, -x\}$
- b. $[x] = [\lfloor x \rfloor, \lfloor x \rfloor + 1)$
- c. $[x] = \{x + 2n\pi : n \in \mathbb{Z}\} \cup \{(2n+1)\pi - x : n \in \mathbb{Z}\}$

5.8. $[f] = \{f + C : C \in \mathbb{R}\}$

7. Counting Measure

7.21. 67,600,000

7.22. 1,679,616

7.23

- a. 21 cards
- b. 324 outcomes
- c. The best hand would be the 5 remaining weapons or the 5 remaining suspects.

7.24. 624

7.25. 7776

7.26. 41,969,002,243,198,805,166,015,625

7.27.

- a. 81
- b. 1080

7.28.

- a. 1024
- b. 120

7.29. 1,048,576

7.30.

- a. 126

b. 35

7.32. 61440

8. Combinatorial Structures

8.25. 720

8.26. 1,965,600

8.27.

a. 40,320

b. 1152

c. 2880

d. 384

8.29.

a. 479,001,600

b. 103,680

c. 4,838,400

8.30.

a. 50,400

b. 9,979,200

c. 34,650

d. 3780

e. 210

8.31. 27,720

8.32.

a. 38,760

b. 13,860

c. 30,800

8.33. 9,777,287,520

8.34.

a. 2,598,960

b. 3744

c. 624

d. 5148

8.35.

a. 635,013,559,600

b. 151,519,319,380

c. 47,079,732,700

d. 11,404,407,300

8.36.

a. 1,913,496

b. 32,427,298,180

8.37. 347,373,600

8.40.

a. 7726

b. 720

8.41.252

8.42. 71,680

8.43. 108,864

8.44. 360,360

8.47.

- a. $\binom{n}{k}$
- b. 7,059,052

8.49.

- a. 210
- b. 56

8.50. 6160

8.51.

- a. 8855
- b. 3876

8.52.

- a. 220
- b. 84

8.53.

- a. Ordered samples with replacement: 10,000
- b. Ordered samples without replacement: 5040
- c. Unordered samples with replacement: 495
- d. Unordered samples without replacement: 210

8.54.

- a. -210
- b. $-\frac{15}{16}$
- c. $\frac{3640}{81}$

8.55.

- a. $\frac{1}{16}$
- b. 70
- c. $-\frac{1}{128}$

8.56.

- a. 365^n .
- b. $365^{\binom{n}{2}}$.

8.57.

- a. 1,625,702,400
- b. 40,320

8.58. 364

8.59.

- a. 252
- b. 126