

**SUPPLEMENT AND CORRIGENDUM**  
**of the book**  
**Attila Nagy, Special Classes of Semigroups**  
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**Preface**

*Page vii, line 2:* ... in which ...

*Page vii, line 2:* ... involves ...

*Page vii, line 24:* ... thirty ...

*Page viii, line 14:* ...camera-ready version ...

**Chapter 1**

*Page 2, line -1, -2, -3:* We remark that if  $N$  denotes a nil semigroup then  $N^1$  will denote the semigroup which can be obtained from  $N$  by the adjunction of an identity element (also in that case when  $|N| = 1$ ).

*Page 3, 24-26:* If  $S$  is a semigroup then let  $S^0$  denote the semigroup  $S \cup \{0\}$  arising from  $S$  by the adjunction of a zero element  $0$  unless  $S$  already has a zero element and  $|S| > 1$ , in which case  $S^0 = S$ .

*Page 6, line 17:*  $\alpha_H$  defined by  $a\alpha_H b$  if and only if  $a = b$  or there is ...

*Page 9, line 8:* semigroups if and only if, for every  $a, b \in S$ , ...

*Page 9, line 11:* ... the assumption  $a \in S^1 b S^1$  implies  $a^n \in S^1 b^2 S^1$

*Page 14, line 7:* Definition 1.36 A 0-simple (a non-trivial simple) semigroup is called completely 0-simple (completely simple) if it contains a primitive idempotent. The trivial semigroup is considered as a completely simple semigroup.

*Page 17, line 13:* ... which determine the ...

*Page 19, line 16:* ... a group  $G$  with a sandwich matrix  $P = (p_{j,i})$  normalized by  $p_{j,i_0} = p_{j_0,i} = e$  for all  $i \in I, j \in J$  and some  $i_0 \in I, j_0 \in J$ , where  $e$  is the identity element of  $G$ , and let  $\mathcal{T}_I$  and ...

*Page 21, line 7:* ... by two letters. (We note that the restriction of the number of variables is not essential.) We suppose ...

*Page 22, line -5:* Conversely, if  $\alpha$  is a ...

*Page 29, line 15:* Hence  $J \subset I$ .

*Page 29, line -6:* ..., by Lemma 2.7

**Chapter 2**

*Page 35, line 6:* assumption  $a \in S^1 b S^1$  implies  $a^n \in S^1 b^2 S^1$  for ...

*Page 35, line -1:*  $(xy)^p \in x^{2^k} S^1$ .

*Page 36, line 1:* Assume  $2^k \geq n$ .

### Chapter 3

Page 57, line 10: ... chain condition.

### Chapter 4

Page 63, line -1:  $a^{2n} = a^{n+1}a^{n-1} = ba^n a^{n-1} = ba^{2n-1} = \dots = b^n a^n$

Page 65, line 14: ... and a positive integer  $m$ .

Page 66, line 12: By induction for  $n$ .

Page 67, line -12:  $ab^n \rho b^{n+1} \rho b^n a$

### Chapter 5

Page 69, line -8 and -9: The right text is the following: We note that, in [106], an  $\mathcal{R}$ -commutative ( $\mathcal{L}$ -commutative,  $\mathcal{H}$ -commutative) semigroup is called a right  $c$ -semigroup (left  $c$ -semigroup,  $c$ -semigroup) if the condition  $x \in S^1$  in Definition 5.1 is satisfied such that  $x \in S$ .

Page 71, line 9: is a commutative congruence. Then, ...

Page 73: Everywhere on page 73, instead of  $\mathcal{R}$ -commutative semigroup,  $\mathcal{L}$ -commutative semigroup,  $\mathcal{H}$ -commutative semigroup should be written right  $c$ -semigroup, left  $c$ -semigroup,  $c$ -semigroup, respectively.

Page 74, line 2:  $\mathcal{L}$ -commutative

### Chapter 6

Page 77, line 7: ... following ...

Page 77, line 15: ... power joined ...

Page 88, line 14: ... sense.

Page 89, line 29: ... rectangular ...

Page 90, line 12: ... relation ...

### Chapter 7

Page 95, line 22: ( $\alpha \leq \delta$ )

Page 95, lines 22, 24, 25: ...  $\mathcal{R}$ -commutative.

Page 95, line -6 and -5: Moreover,  $B \cong E_S$  and so  $B$  is conditionally commutative. As  $E_S$  is a homomorphic image of  $S$ , it is  $\mathcal{R}$ -commutative. Thus  $B$  is  $\mathcal{RC}$ -commutative.

Page 99, line 19: ...  $\Delta$ -semigroup.

Page 100, line 10: By the proof of Lemma 7.4, if  $S_1$  is an abelian group then  $S$  is commutative.

Page 101, line 5:  $|S_0| = 1$  and so  $S = G^0$  is commutative which contradict the assumption for  $S$ .

Page 102, line -8: So  $|R| = 1$  which is a contradiction. Thus the lemma is proved.

Page 103, line 5: ... satisfy ...

Page 107, line -6: (ii)  $S$  is isomorphic to  $R$  or  $R^0$  or  $R^1$ , where  $R$  is a two-element right zero semigroup.

Page 107, lines -3, -2, -1: We remark that Theorem 7.7 and Theorem 7.8 show that the class of  $\mathcal{RC}$ -commutative  $\Delta$ -semigroups form a proper subclass of the class of  $RGC_n$ -commutative  $\Delta$ -semigroups. More precisely, a semigroup is an  $RGC_n$ -commutative  $\Delta$ -semigroup if and only if it is either an  $\mathcal{RC}$ -commutative  $\Delta$ -semigroup or a two-element right zero semigroup with an identity adjoined.

### Chapter 8

Page 109, line 9: described ...

### Chapter 9

Page 119, line 12: left separative ...

Page 122, line 19: ... spined product ...

Page 133: Theorem 9.20 is correct, but using the results of "Nagy, A. and P.R. Jones, *Permutative semigroups whose congruences form a chain*, Semigroup Forum, 69(2004), 446-456", we can give more information: A semigroup  $S$  is a medial  $\Delta$ -semigroup if and only if it satisfies one of the following conditions.

(i)  $S$  is a commutative  $\Delta$ -semigroup.

(ii)  $S$  is isomorphic to either  $R$  or  $R^0$ , where  $R$  is a two-element right zero semigroup.

(iii)  $S$  is isomorphic to the semigroup  $Z = \{0, e, a\}$ , obtained by adjoining to a zero semigroup  $\{0, a\}$  an idempotent element  $e$  that is both a right identity element of  $Z$  and a left annihilator of  $\{0, a\}$ .

(iv)  $S$  is isomorphic to the dual of a semigroup of type (ii) or (iii).

Page 134, line -3: Let  $x \in K$  and ...

Page 135, line 1: ... that  $(c, c^2) \in \alpha$ .

### Chapter 10

it Page 140, line 18: ... spined product ...

Page 144, line 21: ... operation ...

Page 160, line -6: Let  $e_1$  and  $e_2$  be arbitrary elements of  $E$ . It is a matter of checking to see that  $H = \{e_1, e_2\}$  is a normal complex. Let  $\xi$  denote the congruence on  $S$  defined by  $H$ . Assume  $(k, s) \in \xi$  for some  $k \in K_0$  and ...

Page 160, line -2:  $x = 1$ .

Page 161, line 3: ...,  $k$  and  $s$  generate ...

Page 161, line 12: ...,  $k_1g = k_1h$  which ...

Page 162, line 5: ... such ...

*Page 165:* By the new version of Theorem 9.20 (see *Page 133*), Theorem 10.19 can be formulated as follows: A semigroup  $S$  is a right commutative  $\Delta$ -semigroup if and only if it satisfies one of the following conditions.

(i)  $S$  is a commutative  $\Delta$ -semigroup.

(ii)  $S$  is isomorphic to either  $L$  or  $L^0$ , where  $L$  is a two-element left zero semigroup.

(iii)  $S$  is isomorphic to the semigroup  $Z = \{0, e, a\}$ , obtained by adjoining to a zero semigroup  $\{0, a\}$  an idempotent element  $e$  that is both a right identity element of  $Z$  and a left annihilator of  $\{0, a\}$ .

*Pages 165-173:* By the new version of Theorem 10.19, the validity of the last part of Chapter 10 has expired. The assertions are correct, Construction 10.2 gives an example, but the new results about right commutative  $\Delta$ -semigroups show that there are no other examples.

### **Chapter 11**

*Page 177, line 3:* ... abelian ...

### **Chapter 12**

*Page 189, lines 1, 2:* Let  $S = \mathcal{M}(I, G, J; P)$  be a completely simple semigroup expressed ...

*Page 196, line 11:* ... before ...

### **Chapter 13**

*Page 201, line 11:* ... normalized by  $p_{j,i_0} = p_{j_0,i} = e$  ...

*Page 206:* **Theorem 13.9** ([52])

*Page 208, line 17:*  $(\ )\psi_{\alpha,\beta}$  of  $G_\alpha$  into  $G_\beta$ .

*Page 208, line -13:* ... translational hull  $\Omega(K)$  and ...

*Page 210, line 18:* ...  $G_\alpha$  into  $G_\beta$ , ...

### **Chapter 14**

*Page 219, line 3:* ... is satisfied.

*Page 219, line 20:* (vi)  $S$  is a T1 or a T2R or a T2L semigroup.

*Page 221, lines 5 - 8:* If  $|S_1| = 1$  then  $S$  is a T1 semigroup. If  $S_1$  is a two-element left zero semigroup then  $S$  is a T2L semigroup. If  $S_1$  is a two-element right zero semigroup then  $S$  is a T2R semigroup.

*Page 221, line -14:* Theorem 14.11 ([54])  $S$  is a T1 semigroup if and only if ...

*From page 221, line -11 to page 222, line 3:* Proof. As a T1 semigroup is weakly exponential, by Theorem 1.58, it is obvious.

### **Chapter 15**

*Page 226, line 3:* Continuing ...

*Page 231, line 8:* ... generated

*Page 236, line 26:* ... (1, 2)-commutative ...

Page 242, lines 17 - 21: If  $S_a = S$  for all  $a \in S$  then  $a \in S_{a^2}$  which contradicts the fact that  $S$  does not contain idempotent elements. Thus the theorem is proved.

Page 243, line 12: ... obvious.

Page 245, line 9: ...  $N$  or  $N^1$ , where  $N$  is a commutative nil  $\Delta$ -semigroup.

## Chapter 16

Page 247, line 7: ... decomposition ...

Page 250, Theorem 16.2: **Theorem 16.2** ([28]) *If a semigroup  $S$  is  $n_{(2)}$ -permutable ( $n \geq 4$ ) then it is  $(1, 2n - 4)$ -commutative.*

Page 253, line 4: ... which ...

Page 253, line 7: ... homomorphism ...

Page 254, line 2: ... defined in ...

Page 256, line 6: . By Theorem 16.2, Lemma 16.3 and the fact that the  $2_{(2)}$ -permutable semigroup are commutative,  $S$  is  $(1, 2n - 3)$ -commutative.

Page 256, line 13: . Since  $S$  is  $(1, 2n - 3)$ -commutative then,

Page 256, line 24: archimedean ...

Page 257, line 9: . Since an  $n_{(2)}$ -permutable semigroup is  $(1, 2n - 3)$ -commutative

Page 257, Proof of Theorem 16.9: Proof. Using Theorem 16.2, Lemma 16.3 and the fact that the  $2_{(2)}$ -permutable semigroup are commutative, our assertion follows from Theorem 15.13.

Page 257, line 27: ... obvious.

Page 258, Proof of Theorem 16.11: Proof. Using Theorem 16.2, Lemma 16.3 and the fact that the  $2_{(2)}$ -permutable semigroup are commutative, our assertion follows from Theorem 15.15.

Page 258, line 8: ...  $N$  or  $N^1$ , where  $N$  is a commutative nil  $\Delta$ -semigroup.

Page 258, Proof of Theorem 16.12: Proof. Using Theorem 16.2, Lemma 16.3 and the fact that the  $2_{(2)}$ -permutable semigroup are commutative, our assertion follows from Theorem 15.16.