Komplex függvénytan

$x^2 = -1$

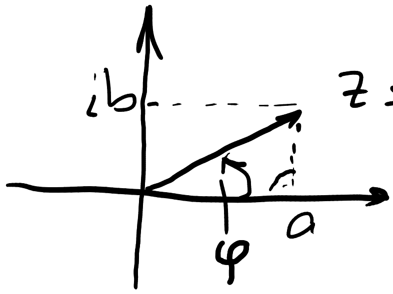
Komplex számok:

$a+bx \mid \underline{x^2} = (x^2+1) + \underline{(-1)}$

$\mathbb{C} = \{a+ib \mid a;b \in \mathbb{R}\} \cong \mathbb{R}[x]/(x^2+1) \cong$

$\underline{i^2=-1}$

$\cong \{M \in \mathbb{R}^{2\times 2} \mid M$ forgatva nyújtás$\}$

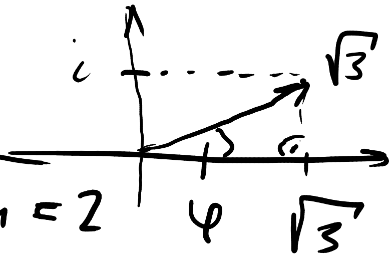


$z = a+ib \rightarrow$ $Re(a+ib) = a \in \mathbb{R}$

$Im(a+ib) = b \in \mathbb{R}$

$|z| = r$

φ z szöge



$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

$tg\, \varphi = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \varphi = \frac{\pi}{6}$

- algebrai alak: $z = a+ib = r\cos\varphi + ir\sin\varphi$
- trigonometrikus: $z = r\cdot(\cos\varphi + i\sin\varphi)$
- exponenciális alak: $z = r\cdot e^{i\varphi}$

Euler-formula $\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi}$ $\varphi \in \mathbb{R}$

$$z_1 \cdot z_2 = r_1 r_2 (\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$$
$$= r_1 r_2 \cdot e^{i(\varphi_1+\varphi_2)} = r_1 e^{i\varphi_1} \cdot r_2 e^{i\varphi_2} =$$
$$= r_1 r_2 e^{i\varphi_1 + i\varphi_2}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\varphi_1-\varphi_2)} \quad ; \quad z^n = r^n e^{n\cdot i\varphi}$$

① $$\frac{1}{\sqrt{3}-i} = \frac{1}{\sqrt{3}-i}\cdot\frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{\sqrt{3}+i}{(\sqrt{3}-i)(\sqrt{3}+i)} =$$

$$= \frac{\sqrt{3}+i}{4} = \underbrace{\frac{\sqrt{3}}{4}}_{Re(z)} + \underbrace{\frac{1}{4}}_{Im(z)}\cdot i$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(\sqrt{3})^2 - (i)^2 = 3 - (-1) = 4$$

$$\bar{z} = \boxed{\overline{x+iy} \overset{def}{=} x - iy}$$; ② $\overline{\underbrace{\sqrt{3}-1}_{x}\ \underbrace{+0\cdot i}}= \sqrt{3}-1$

$$\overline{\sqrt{5}+3i} = \sqrt{5} - 3i$$

$(\cancel{a+\sqrt{3}b})$

Komplex függvények

$$f: \mathbb{C} \to \mathbb{C}, \quad x+iy \mapsto f(x+iy)$$

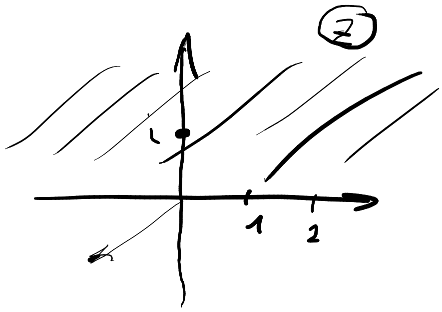
$\mathrm{Dom}(f) \subseteq \mathbb{C}$

$$f(x+iy) = \underbrace{u(x,y)}_{\substack{Re(f)\\ :\mathbb{R}^2 \to \mathbb{R}}} + i\cdot \underbrace{v(x,y)}_{\substack{Im(f)\\ :\mathbb{R}^2 \to \mathbb{R}}}$$

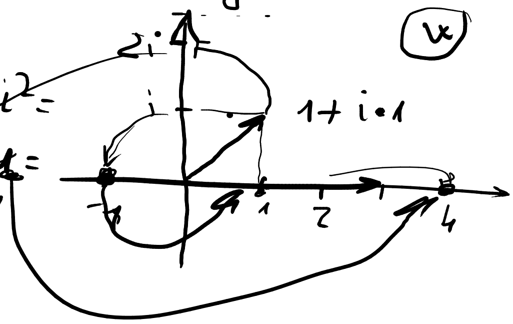
② $f(z) = z^2$ $\quad$ $Re f,\ Im f = ?$

Trükk: $\overset{\in\mathbb{R}}{x} + i\overset{\in\mathbb{R}}{y}$;

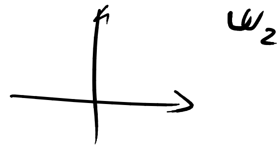
$$(x+iy)^2 = x^2 + 2xiy + \underbrace{(iy)^2}_{-y^2} = \underbrace{x^2-y^2}_{u(x,y)} + i\underbrace{2xy}_{v(x,y)}$$



$(1+i)^2 =$
$= 1 + 2i + i^2 =$
$= 1 + 2i - 1 =$
$= 2i$



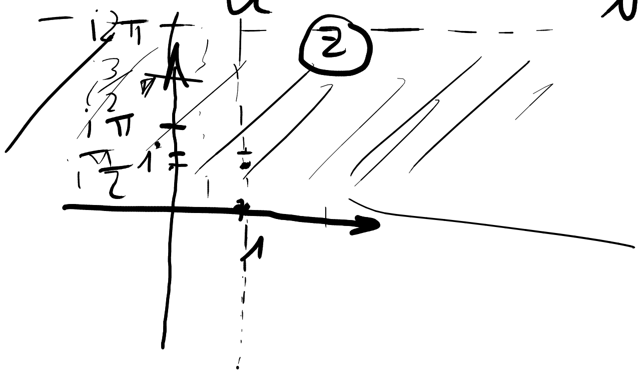
$|1+i| = ?$
$= \sqrt{2}$
$\sqrt{1^2+1^2}$
$i^2 = -1$

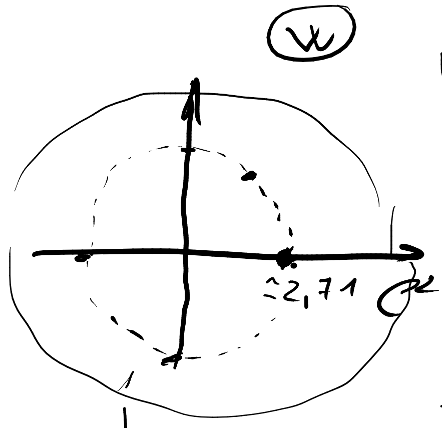


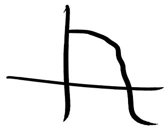
③ $x+iy \xrightarrow{f} e^{x+iy}$ $\qquad$ $\mathrm{Re}\, f = ?$ $\mathrm{Im}(f) = ?$

$$e^{x+iy} = e^{x} \cdot e^{iy} = e^{x}(\cos y + i \sin y) =$$

$$= \underbrace{e^{x}\cos y}_{u} + i \cdot \underbrace{e^{x}\sin y}_{v}$$







$x+iy$ $\qquad$ $e^{z} = e^{x}(\cos y + i \sin y)$

$\cos 1$

Elemi függvények (exp, pol, trig, log. hyp)

$$\sin z \stackrel{def}{=} \frac{e^{iz} - e^{-iz}}{2i}, \quad \mathrm{sh}\, z = \frac{e^{z} - e^{-z}}{2}$$

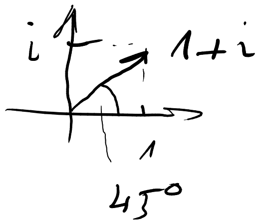
$$\cos z \stackrel{def}{=} \frac{e^{iz} + e^{-iz}}{2}, \quad \mathrm{ch}\, z = \frac{e^{z} + e^{-z}}{2}$$

$$\mathrm{Ln}\, z \stackrel{def}{=} \underbrace{\ln(r)}_{\in \mathbb{R}} + i \cdot \varphi + k \cdot 2\pi i$$

$z = re^{i\varphi}$

④ $e^{z} = 1+i \quad / \mathrm{Ln}$

$$z = \mathrm{Ln}(1+i) = \ln(\sqrt{2}) + i \cdot \frac{\pi}{4} + k \cdot 2\pi i$$



$r = \sqrt{2}$

$\frac{\pi}{4} = \varphi$

k egész szám (levelek száma)