

Introduction to Dynamical Systems and Chaos

take home final exam - corrected version

Imre Péter Tóth, fall 2005

As an exam, please solve the following exercises. They are meant to be easier than the midterm exercises were, because your time is limited, but they cover a bigger part of the material. Please work on your own. Of course, using the book is allowed.

1. Consider the function $f_a(x) = ax(1 - x)$ where a is some real parameter. Show three different values of a where the dynamical system defined by f_a exhibits three greatly different kinds of behaviour.
2. Consider the two-dimensional map $f(x; y) = (x^2 - x + 2y - 1; 2x - 2x^3 + y)$. Check that the point $(1; 0)$ is periodic, and determine the stability of its orbit.
3. Consider the Smale Horseshoe map as shown in Figure 5.16 of the book.
 - (a) Is there an orbit, which never enters the region marked as V_L in Figure 5.16 ?
 - (b) Is there an orbit, which never enters the region marked as V_L in Figure 5.16, but which is dense in the rest of the invariant Cantor set?
 - (c) Is there an orbit, which never enters the region marked by 1 in Figure 5.17, but which is dense in the rest of the invariant Cantor set?
4. Take a chess-board, and draw a smaller chess-board (of size $\frac{1}{8}$ of the original) instead of every black field. (White fields remain entirely white.) Then repeat this step infinitely many times with the small black squares you have after the previous step. What is the box dimension on the set of points which remain black forever?
5. Solve Exercise 7.12 from the book.