Midterm II.

Mathematics A3 in English for Civil Engineering students

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1. (6 points) Joe is subscribed for three TV channels, called "Allergy Channel", "Boredom Channel" and "Catastrophe Channel". On Allergy Channel, 20% of the broadcast time is filled with commercials. This proportion is 30% for Boredom Channel and 40% for Catastrophe Channel. This morning Joe turned on his TV and switched to one of these channels at random. He sadly saw that a commercial was running. What is the probability that he was watching Allergy Channel?

2. (6 points) Joe has three chances to pass his Math exam. Before the first exam he spends 3 hours perparing. If he fails, then for the second attempt he only prepares 2 hours. If he fails again, he only prepares 1 hour for the last attempt. The success probability for each exam is proportional to the time spent with preparation: it is 30% for the first, while only 20% for the second and 10% for the third attempt.

   What is the probability that he will pass on (exactly) the third exam?

3. (6 points) We roll two fair dice – red and blue. Let us define the following events:

   - A: the red die shows 6
   - B: the blue die shows an odd number
   - C: the sum of the two numbers shown is 7.

   (a) Are A and B independent?

   (b) Are A and C independent?

   (c) Are B and C independent?

   (d) Are A, B and C mutually independent?

4. (6 points) On an exam of "Fitness Studies", Joe has to answer 10 multiple-choice questions: for each question he has to choose the (only) correct answer from a list of 4. For each question (independently of the other questions) he knows the correct answer with probability 60%. If he doesn't know, he chooses an answer out of the 4 at random. (So he can still be lucky and hit the correct one.) In order to get the best grade (5), he needs to get at least 9 answers right.

   (a) What is the probability that he marks the correct answer for question 1?

   (b) What is the probability that he gets a 5?

5. (6 points) The e-mail box of Joe has a good spam filter: it filters out the vast majority of spam that is being sent to him. However, some of the spam manage to pass through, so he gets on the average 1 message per week in which they ask him to send his credit card number. What is the probability that Joe will get at least two such messages tomorrow?

Solutions

1. Application of Bayes Theorem. Notation: $A := \{\text{Allergy}\}$, $B := \{\text{Boredom}\}$, $C := \{\text{Catastrophe}\}$, $D := \{\text{commercial}\}$. So

$$P(A|D) = \frac{P(A)P(D|A)}{P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)} = \frac{\frac{1}{3}\frac{2}{10}}{\frac{1}{3}\frac{2}{10} + \frac{1}{3}\frac{3}{10} + \frac{1}{3}\frac{4}{10}} = \frac{2}{9}.$$

2. $P(\text{passes 3-rd}) = P(\text{fails first})P(\text{fails second}|\text{fails first})P(\text{passes third}|\text{fails first two}) = \frac{7}{10}\frac{8}{10}\frac{1}{10} = 0.056$

3. $P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{2}$ obviously. $P(C) = P(\{16, 25, 34, 43, 52, 61\}) = \frac{6}{36} = \frac{1}{6}$.
$P(AB) = P(\{61, 63, 65\}) = \frac{3}{36} = \frac{1}{12}$, $P(AC) = P(\{61\}) = \frac{1}{36}$, $P(BC) = P(\{61, 43, 25\}) = \frac{3}{36} = \frac{1}{12}$.
So

   (a) YES, because $P(AB) = P(A)P(B)$.

   (b) YES, because $P(AC) = P(A)P(C)$.

   (c) YES, because $P(BC) = P(B)P(C)$.

   (d) NO, because they are pairwise independent (see the previous three points), but

$$P(ABC) = P(\{61\}) = \frac{1}{36} \neq P(A)P(B)P(C).$$

4. (a) $P(\text{correct}) = P(\text{he knows}) + P(\text{doesn't know})P(\text{he's lucky}|\text{doesn't know}) = \frac{6}{10} + \frac{4}{10}\frac{1}{4} = \frac{7}{10}$.

   (b) Let X denote the number of correct answers marked, so $X \sim \text{Bin}(n, p)$ with $n = 10$ and $p = \frac{7}{10}$. Now

$$P(\text{he gets a 5}) = P(X \geq 9) = \binom{10}{9}\left(\frac{7}{10}\right)^9\left(\frac{7}{10}\right)^1 + \left(\frac{7}{10}\right)^{10} = 0.149$$

5. Let X denote the number of "credit card spam" he gets tomorrow. $X \sim \text{Poi}(\lambda)$, because many independent attempts occur with small succes probability. $\lambda = E(X) = \frac{1}{7}$, because 1 spam per week is $\frac{1}{7}$ spam per day (on the average). So

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-1/7} - e^{-1/7}\frac{1}{7} = 0.0093$$