Stochastic differential equations
TU Budapest, spring semester 2019
Imre Péter Tóth
Second makeup midterm exam, 21.05.2019
Working time: 90 minutes

In this exercise sheet, "Brownian motion" is used as a synonym of "Wiener process", and $B(t)$, $B_1(t)$, $B_2(t)$, denote independent standard 1-dimensional Brownian motions.

1. (6 points) For which $c \in \mathbb{R}$ is $\mathbb{P}(\exists t > 0 : B(t) = c + t) = 1$? Justify your statement.

2. (6 points) Calculate the following probabilities:

   a.) $\mathbb{P}(\exists t \in [0,2] : B(t) > 2)$

   b.) $\mathbb{P}(\exists t \in (1,2) : B(t) = B(1))$

   c.) $\mathbb{P}(\exists t \in [0,2] : |B(t)| > t)$

3. (6 points) Let $\lambda, s, t > 0$. How much is the expectation

$$\mathbb{E}\left(e^{\lambda[B_1(t+s)+B_2(t+s)-B_1(t)-B_2(t)]}\right)?$$

4. (6 points) Calculate the probabilty that the process $X(t) := B(t) + 50t$ hits the point 100 before hitting -1. *(Hint: with λ chosen correctly, $e^{\lambda X(t)}$ is a martingale.)*

5. (6 points) For which functions $f : [0,\infty) \to \mathbb{R}$ is the process $X(t) := f(t)B\left(\frac{1}{1+t}\right)$ a martingale?