

$$\textcircled{1} \lim_{x \rightarrow 0+} \frac{\int_0^x (\operatorname{ctg} t)^t dt}{\sin x} \stackrel{L'H}{=} \lim_{x \rightarrow 0+} \frac{(\operatorname{ctg} x)^x}{\cos x} = \lim_{x \rightarrow 0+} (\operatorname{ctg} x)^x = \lim_{x \rightarrow 0+} e^{x \ln \operatorname{ctg} x} = e^0 = 1$$

$$\frac{d}{dx} \left( \int_0^x (\operatorname{ctg} t)^t dt \right) = (\operatorname{ctg} x)^x$$

$$\lim_{x \rightarrow 0+} x \frac{\ln(\operatorname{ctg} x)}{\frac{1}{x}} = \lim_{x \rightarrow 0+} \frac{\ln(\operatorname{ctg} x)}{\frac{1}{x}} = \lim_{x \rightarrow 0+} \frac{\frac{1}{\operatorname{ctg} x} \cdot -\frac{1}{\sin^2 x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0+} \frac{1}{(\operatorname{ctg} x)} \cdot \frac{1}{\left(\frac{\sin x}{x}\right)^2} = 1$$

Megj. Többben hivatalosan arra h. ha  $\lim_{x \rightarrow x_0} f(x) = a$ ,  $\lim_{x \rightarrow x_0} g(x) = b$ , akkor

ilyen esetben nem tanítottuk, ís nem is igaz minden esetben.

Mihez igaz?

All Ha  $f(x) > 0$   $x_0$ -nél többet (x<sub>0</sub>-hoz elég közel) és  $a > 0$ , akkor

$$\lim_{x \rightarrow x_0} f(x)^{g(x)} = a^b$$

$$\text{B.t. } f(x)^{g(x)} = e^{g(x) \ln f(x)} \xrightarrow{x \rightarrow x_0} e^{b \ln a} = a^b$$

minél hibás n>0-ban teljes.

Meg. Itt fontos, hogy  $f(x) > 0$  és  $\lim_{x \rightarrow x_0} f(x) = a > 0$ , különben az áll nem alk-ható.

Igy  $\lim_{x \rightarrow 0+} (\operatorname{ctg} x)^x$  minden esetben, hiszen  $\lim_{x \rightarrow 0+} \operatorname{ctg} x = +\infty$

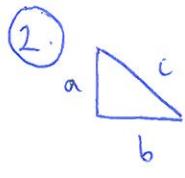
$(\operatorname{ctg} x)^x = \frac{(\cos x)^x}{(\sin x)^x}$ , ít  $(\cos x)^x \xrightarrow{x \rightarrow 0+} 1^0 = 1$  minden esetben, de  $\lim_{x \rightarrow 0+} (\sin x)^x$  -nál az áll. nem mond semmit

$$\text{minél } \lim_{x \rightarrow 0+} \sin x = 0 = a$$

$$\lim_{x \rightarrow 0+} \left( e^{-\frac{1}{x^2}} \right)^x = \lim_{x \rightarrow 0+} e^{-\frac{1}{x}} = 0$$

$$\lim_{x \rightarrow 0+} \left( e^{-\frac{1}{x^2}} \right)^{-x} = \lim_{x \rightarrow 0+} e^{\frac{1}{x}} = +\infty$$

azaz  $0^\infty$  típusú hibák esetében a gyüjtőtől nem lehetős, h. 1-ig a "szemben".



Ha pl.  $a+c=10$ , akkor  $c=10-a$ ,  $b=\sqrt{c^2-a^2}=$   
 $=\sqrt{(10-a)^2-a^2}=\sqrt{100-20a}$

Igy  $t=\frac{a \cdot b}{2}=\frac{a \sqrt{100-20a}}{2}$

(itt  $0 < a < 5$ , hiszen  $0 < a < c$  is  $a+c=10$ )

Hasonlós elminálás:  $t=t(a)=\frac{a \sqrt{100-20a}}{2}$  ugyanúttal maximális, ahol

$f(a)=2t^2(a)=a^2(100-20a)$  maximális, mivel  $f'(a)>0$ .

$f(a)-t$  csaknál hőnyegből deriválni, mint  $t(a)-t$ !

Hn  $f(a)=a^2(100-20a)$ ,  $0 < a < 5$  az a-ban maximális, ahol  $\partial t f'(a)=0$

$f'(a)=200a-60a^2=\frac{20a}{3}(10-\frac{3}{2}a)=0 \Leftrightarrow a=0$  v.  $a=\frac{10}{3} \in (0,5)$ .

Kerülés  $a=\frac{10}{3}$  valóban maximumoly?

Válasz Igen, mert  $f(a)$  jojt fr-nél  $[0,5]$  hossz. szakaszon van max. hely, és a 0-ban, 5-ben lehets v. ott, ahol  $f'(a)=0$ .

Dz  $f(0)=f(5)=0$ ,  $f(\frac{10}{3})>0$ , így  $\frac{10}{3}$ -ban van a max. hely.

(3.) 1. Mo  $m=\int_0^1 1 \arcsin x \, dx = [\arcsin x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}} \, dx = 1 \arcsin 1 - 0 + \left[ \frac{\sqrt{1-x^2}}{2} \right]_0^1 = \frac{\pi}{2} - 1$

$M_y=\int_0^1 x \arcsin x \, dx = \left[ \frac{x^2}{2} \arcsin x \right]_0^1 - \int_0^1 \frac{x^2}{2} \frac{1}{\sqrt{1-x^2}} \, dx = \frac{\pi}{4} - \int_0^{\pi/2} \frac{\sin^2 t}{2} \frac{1}{\sqrt{\cos^2 t}} \cos t \, dt$

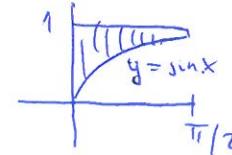
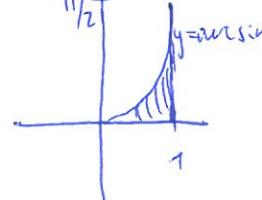
$x = \sin t$   
 $\frac{dx}{dt} = \cos t$

$\text{Helyettesítés}$   
 $x=0 \rightarrow t=0$   
 $x=1 \rightarrow t=\frac{\pi}{2}$

$\frac{dx}{dt}$

$$= \frac{\pi}{4} - \frac{1}{2} \int_0^{\pi/2} \sin^2 t \, dt = \frac{\pi}{4} - \frac{1}{2} \int_0^{\pi/2} \frac{1-\cos 2t}{2} \, dt = \frac{\pi}{4} - \frac{1}{2} \left[ \frac{t}{2} + \frac{\sin 2t}{4} \right]_0^{\pi/2} = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8}$$

Igy  $s_x = \frac{M_y}{m} = \frac{\frac{\pi}{8}}{\frac{\pi}{2}-1}$



2. Mo A megadott tart. szögpontjainak x hossza =  $\{0 \leq x \leq \frac{\pi}{2}, \sin x \leq y \leq 1\}$  tart. szögpontjainak y hossza, amit

$$\frac{\int_0^{\pi/2} \frac{1-(\sin x)^2}{2} \, dx}{\int_0^{\pi/2} 1-\sin x \, dx} = \frac{\frac{\pi}{4} - \frac{1}{2} \int_0^{\pi/2} \frac{1-\cos 2x}{2} \, dx}{\frac{\pi}{2} + \cos \frac{\pi}{2} - \cos 0} = \frac{\frac{\pi}{4} - \frac{\pi}{8} + \left[ \frac{\sin 2x}{8} \right]_0^{\pi/2}}{\frac{\pi}{2} + 0 - 1} = \frac{\frac{\pi}{8}}{\frac{\pi}{2}-1}$$

(4)

$$\begin{aligned} f(x) &= \sqrt[4]{x} \\ f'(x) &= \frac{1}{4} x^{-\frac{3}{4}} \\ f''(x) &= -\frac{3}{16} x^{-\frac{7}{4}} \\ f'''(x) &= \frac{21}{64} x^{-\frac{11}{4}} \\ &\vdots \end{aligned}$$

$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(16)}{k!} (x-16)^k$$

$$\begin{aligned} x_0 &= 16 & f(x_0) &= f(16) = \sqrt[4]{16} = 2 \\ f'(x_0) &= f'(16) = \frac{1}{4} 2^{-3} = \frac{1}{2^5} = \frac{1}{32} \\ \frac{f''(x_0)}{2!} &= \frac{f''(16)}{2} = \frac{-3}{32} 2^{-7} = \frac{-3}{2^{12}} \end{aligned}$$

$$\sqrt[4]{17} = f(17) \approx T_n(17) = \sum_{k=0}^n \frac{f^{(k)}(16)}{k!} \underbrace{(17-16)^k}_1$$

$$\sqrt[4]{17} - T_n(17) = \frac{f^{(n+1)}(t)}{(n+1)!} (17-16)^{n+1} = \frac{f^{(n+1)}(t)}{(n+1)!}, \text{ wobei } 16 \leq t \leq 17.$$

(Lagrange-Mittelwert)

$$\max_{16 \leq t \leq 17} \left| \frac{f^{(n+1)}(t)}{(n+1)!} \right| = \left| \frac{f^{(n+1)}(16)}{(n+1)!} \right|, \text{ weil } t \text{ konstant} \oplus.$$

Kontrollieren Sie, ob  $\left| \frac{f^{(n+1)}(16)}{(n+1)!} \right| \leq 10^{-3}$ . Wenn ja, dann ist  $n=1$  ausreichend,

$$\left| \frac{-3}{2^{12}} \right| < \frac{4}{2^{12}} = \frac{1}{2^{10}} = \frac{1}{1024} < 10^{-3}.$$

Also  $\sqrt[4]{17} \approx f(16) + f'(16)(17-16) = 2 + \frac{1}{32} = \underline{\underline{\frac{65}{32}}} \text{ möglich}$

\*  $\sqrt[4]{15} \approx f(16) + f'(16)(15-16) = 2 - \frac{1}{32} = \underline{\underline{\frac{63}{32}}} \text{ möglich}$

Wäre es falsch möglich:  $\max_{15 \leq t \leq 16} \left| \frac{f''(t)}{2!} (15-16)^2 \right| = \max_{15 \leq t \leq 16} \frac{3}{32} t^{-\frac{7}{4}} = \underline{\underline{\frac{3}{32} 15^{-\frac{7}{4}}}}$

$\left( \frac{\sum}{32} 16^{-\frac{7}{4}} \right)$

(5.)  $S = \int_0^{\pi/4} \sqrt{1 + [\tan(\cos x)]^2} dx = \int_0^{\pi/4} \sqrt{1 + \left(\frac{-\sin x}{\cos x}\right)^2} dx = \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx = \star$

Innen schließen lebt fortan

1. fkt  $\star = \int_0^{\pi/4} \sqrt{\frac{1}{\cos^2 x}} dx = \int_0^{\pi/4} \frac{1}{\cos x} dx = \int \frac{1}{1-t^2} \frac{2}{1+t^2} dt$

$\begin{cases} \tan\left(\frac{x}{2}\right) = t \\ \text{hypotenuse} \end{cases}$ , a "mindest mögliche" trig  
hypotenuse, nem fkt. a legenwölb

$= \int \frac{2}{1-t^2} dt = \int \frac{2}{(1-t)(1+t)} dt = \int \frac{a}{1-t} + \frac{b}{1+t} dt = \int \frac{1}{1-t} + \frac{1}{1+t} dt =$

$a(1+t) + b(1-t) = 2$

$\begin{array}{l|l} t=0 & a+b=0 \rightarrow a=b \\ t=1 & a+b=2 \rightarrow \downarrow \\ & a=b=1 \end{array}$

$= \left[ -\ln|1-t| + \ln|1+t| \right] = \left[ \ln \left| \frac{1+\tan\left(\frac{x}{2}\right)}{1-\tan\left(\frac{x}{2}\right)} \right| \right]_0^{\pi/4} = \ln \left| \frac{1+\tan\left(\frac{\pi}{8}\right)}{1-\tan\left(\frac{\pi}{8}\right)} \right| - \ln 1$

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2. fkt eignen sich bspw.

$\star = \int_0^1 \sqrt{1+t^2} \frac{1}{1+t^2} dt = \int_0^1 \frac{1}{\sqrt{1+t^2}} dt = [\arsh t]_0^1 = \arsh 1 - \arsh 0$

$\begin{aligned} \tan x &= t \\ x &= \arctan t \\ \frac{dx}{dt} &= \frac{1}{1+t^2} \end{aligned}$

$\boxed{\arsh x = \ln(x + \sqrt{x^2+1})}$

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(6.)  $(x^3+x)y' = y \ln y$  mit v.l.a.nthato' DE.

$\bullet (x^3+x) \frac{dy}{dx} = y \ln y$  (dann lebt weiter ylog-nal, da  $\neq 0$ )

$\int \frac{1}{y \ln y} dy = \int \frac{1}{x^3+x} dx = \int \frac{1}{x(x^2+1)} dx = \int \frac{a}{x} + \int \frac{bx+c}{x^2+1} dx = \int \frac{1}{x} - \frac{1}{x^2+1} dx$

$\ln|\ln y| = \ln|x| - \ln(x^2+1) + C_1$

$|\ln y| = \frac{1}{e^{C_1}} e^{\frac{1}{x^2+1}}$

$\ln y = C \frac{x}{\sqrt{x^2+1}}$

$y = e^{\frac{C \frac{x}{\sqrt{x^2+1}}}{C \in \mathbb{R}}}$  obwohl  $y \geq 1$   
wenn  $C=0$ -val

$\begin{array}{l|l} x^2 & a+b=0 \rightarrow b=-1 \\ x & c=0 \\ 1 & a=1 \end{array}$

$y(2)=1 : 1 = e^{C \frac{2}{\sqrt{5}}} \rightsquigarrow C=0$

$y(1)=e : e = e^{C \frac{1}{2}} \rightsquigarrow C=\frac{1}{2}$