Analízis [illegible] és [illegible] 2. zh. 2017. nov. 21. [illegible]

1.

$$\ln(x^3-7y^2)-xy = 2\cos(\pi y)$$

$$\ln(8-7) - 2\cdot 1 = 2\cos(\pi)$$

$$\underbrace{\ln 1}_{0} - 2 = -2 \quad ✓$$ (0,1)

$$\frac{d}{dx}: \quad \frac{3x^2-14y\cdot y'}{x^3-7y^2} - (y+xy') = -2\sin(\pi y)\,\pi y'$$ (1,4)

$(x,y) = (2,1)$ pontban:

2,5p

$$\frac{12-14y'}{8-7} - (1+2y') = -2\underbrace{\sin\pi}_{0}\cdot\pi\cdot y'$$ (0,4)

$$11-16y' = 0$$

$$y' = \frac{11}{16} = m$$ (0,3)

$$y-1 = \frac{11}{16}(x-2)$$ (0,3)

2.

$$f(x) = \frac{e^x}{x+1} \neq 0, \quad D_f = \mathbb{R}\setminus\{-1\}$$

x		-1	
f	$\ominus$	$\nexists$	$\oplus$

(0,2)

$$f'(x) = \frac{e^x(x+1)-e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$ (0,2)

x		-1		0	
f'	$\ominus$	$\nexists$	$\ominus$	0	$\oplus$
f	↘		↘	lok. min.	↗

(0,6)

4p

$$f''(x) = \frac{[e^x+xe^x](x+1)^2 - xe^x\,2(x+1)}{(x+1)^4} = \frac{e^x[(x+1)^2-2x]}{(x+1)^3} = \frac{e^x(x^2+1)}{(x+1)^3}$$ (0,6)

x		-1	
f''	$\ominus$	$\nexists$	$\oplus$
f	∩	$\nexists$	∪

(0,4)

$$\lim_{x\to\infty} f(x) \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to+\infty}\frac{e^x}{1} = +\infty$$ (0,2)

$$\lim_{x\to-\infty}\frac{e^x}{x+1} = \text{"}\frac{0}{-\infty}\text{"} = 0$$ (0,2)

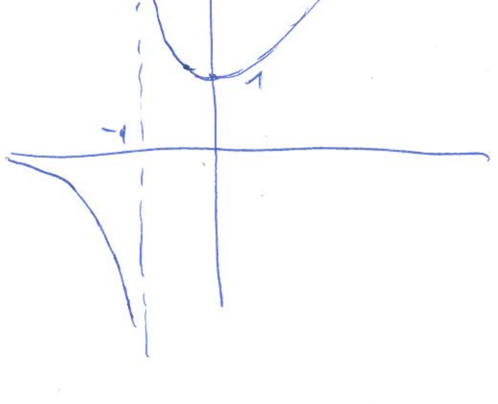
$y=0$ $x\to-\infty$-ben vízszintes aszimptota.

$$\lim_{x\to-1+}\frac{e^x}{x+1} = +\infty$$ (0,2)

$$\lim_{x\to-1-}\frac{e^x}{x+1} = -\infty$$ (0,2)

$x=-1$ függőleges asz.

$$\lim_{x\to\infty}\frac{e^x}{x(x+1)} = 0 \quad (L'H\ 2\times) \Rightarrow \text{ferde aszimptota nincsen.}$$ (0,2)



(1)

$$R_f = \mathbb{R}\setminus[0,1) = (-\infty,0)\cup[1,+\infty)$$

3.

$\ln 4 - \ln 3 = \ln \frac{4}{3}$ (0,1)

$f(x) = \ln x$ (0,1), $x_0 = 1$ (0,1)

$f'(x) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f^{IV}(x) = \frac{-6}{x^4}$ (0,2)

$f(1) = 0$

$f'(1) = 1$

$f''(1) = -1$

$f'''(1) = 2$

$f^{IV}(1) = -6$ (0,2)

$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k = \sum_{k=1}^{n} \frac{f^{(k)}(1)}{k!}(x-1)^k$ (0,1)

$f(x) - T_n(x) = \frac{f^{(n+1)}(t)}{(n+1)!}(x-x_0)^{n+1}$ (0,1)

Most $\ln\frac{4}{3} = f\left(\frac{4}{3}\right) \approx T_n\left(\frac{4}{3}\right)$ (0,1), $n = ?$ kell $\left|\frac{f^{(n+1)}(t)}{(n+1)!}(x-x_0)^{n+1}\right| \le 10^{-2}$ (0,1)

itt $x = \frac{4}{3}$, $x_0 = 1$, $1 \le t \le \frac{4}{3}$ (0,2) $\left|\frac{f^{(n+1)}(t)}{(n+1)!}\left(\frac{1}{3}\right)^{n+1}\right| \le 10^{-2}$ (0,1)

2,3p

Itt $\max\limits_{1 \le t \le \frac{4}{3}} |f^{(n+1)}(t)| = |f^{(n+1)}(1)|$ (0,3)

$\frac{f^{(3)}(1)}{3!}\left(\frac{1}{3}\right)^3 = \frac{2}{6}\cdot\frac{1}{3^3} = \frac{1}{81}$ (0,2) még nem jó

$\left|\frac{f^{(4)}(1)}{4!}\left(\frac{1}{3}\right)^4\right| = \left|\frac{-6}{24}\cdot\frac{1}{3^4}\right| = \frac{1}{4}\cdot\frac{1}{81} \le 10^{-2}$ ✓ (0,2)

Így $n=3$ már jó (0,2)

0,7p

$T_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f'''(1)}{6}(x-1)^3$ (0,1)

$T_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$ (0,3)

$T_3\left(\frac{4}{3}\right) = \frac{1}{3} - \frac{1}{2}\cdot\frac{1}{9} + \frac{1}{3}\cdot\frac{1}{27} = \frac{1}{3} - \frac{1}{18} + \frac{1}{81}$ (0,3)

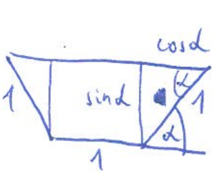
Σ 4p

1p

$T_3\left(\frac{2}{3}\right) = \frac{-1}{3} - \frac{1}{2}\cdot\frac{1}{9} + \frac{1}{3}\left(-\frac{1}{3}\right)^3 = \frac{-1}{3} - \frac{1}{18} - \frac{1}{81}$ (0,2)

Itt a hiba max: $\max\limits_{\frac{2}{3} \le t \le 1}\left|\frac{\frac{-6}{t^4}}{4!}\left(\frac{2}{3}-1\right)^4\right| = \frac{1}{4}\,\frac{1}{3^4}\,\frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{2^6} = \frac{1}{64}$ (0,3) (0,3) (0,2)

4.



Feltehető, h. a deszkák szélessége 1 egység.

$t(\alpha) = 1\cdot\sin\alpha + 2\,\frac{\sin\alpha\cdot\cos\alpha}{2} = \sin\alpha + \sin\alpha\cdot\cos\alpha$ (0,5)

$t'(\alpha) = \cos\alpha + \cos^2\alpha - \sin^2\alpha$ (0,4) $= \cos\alpha + \cos^2\alpha - (1-\cos^2\alpha) = 2\cos^2\alpha + \cos\alpha - 1 = 0$ (0,6)

3p

$\cos\alpha = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$ → -1: $\alpha = \pi$ ∅; → $\frac{1}{2}$ (0,5): $\alpha = 60°$

Deszkák által bezárt szög: $120°$ (0,5)

(0,5) Ez valóban max hely, mivel $t(0) = t(\pi) = 0$

t $[0,\pi]$-n folyt $\Rightarrow \exists$ max hely, vagy $\alpha = 0$, $\alpha = \pi$ v. $\alpha = \frac{\pi}{3}$-ban lehet, és nem 0, ill π-ben van.

5a) $\int_0^{\pi} x \cos x \sin x \, dx = \int_0^{\pi} x \frac{\sin 2x}{2} dx = \left[x \cdot \frac{-\cos 2x}{4} \right]_0^{\pi} - \int_0^{\pi} 1 \cdot \frac{-\cos 2x}{4} dx =$ (0,25) (0,5)

$f \quad g' \qquad f \quad g$

1,25 p

$= \pi \cdot \frac{-\cos(2\pi)}{4} - 0 + \left[\frac{\sin 2x}{8} \right]_0^{\pi} = \frac{-\pi}{4} + \frac{\sin 2\pi - \sin 0}{8} = \frac{-\pi}{4}$ (0,2) (0,2) (0,1)

1,25 p $\int_1^e \frac{\sqrt[3]{\ln x}}{x} dx = \int_1^e f^{1/3} \cdot f' = \left[\frac{f^{4/3}}{4/3} \right]_1^e = \left[\frac{(\ln x)^{4/3}}{4/3} \right]_1^e = \frac{3}{4} \left[(\ln e)^{4/3} - (\ln 1)^{4/3} \right] = \frac{3}{4}$ (0,3) (0,6) (0,15) (0,2) (0,35)

1,25 p $\int_0^1 \frac{x}{1+x^4} dx = \frac{1}{2} \int_0^1 \frac{2x}{1+(x^2)^2} dx = \frac{1}{2} \left[\operatorname{arctg}(x^2) \right]_0^1 = \frac{1}{2} \left(\operatorname{arctg} 1 - \operatorname{arctg} 0 \right) = \frac{\pi}{8}$ (0,5) (0,5) (0,25)

1,25 p $\int \left(\frac{x}{x+1} \right)^2 dx = \int 1 - \frac{2x+1}{x^2+2x+1} dx = x - \int \frac{2x+2}{x^2+2x+1} + \int \frac{1}{(x+1)^2} dx =$ (0,3) (0,55)

$= x - \ln(x^2+2x+1) - \frac{1}{x+1} + C = x - 2\ln|x+1| - \frac{1}{x+1} + C$

(0,4)

vagy

$\int \left(\frac{x}{x+1} \right)^2 dx = \int \frac{(t-1)^2}{t} dt = \int 1 - \frac{2}{t} + \frac{1}{t^2} dt = t - 2\ln|t| - \frac{1}{t} =$

$x+1 = t, \quad x = t-1 \qquad = x+1 - 2\ln|x+1| - \frac{1}{|x+1|} + C$

$\frac{dx}{dt} = 1$