$$59x^2+24xy+66y^2-430x-240y+725=0$$

$$\underline{\underline{A}}=\begin{bmatrix}59 & 12 & -215\\ 12 & 66 & -120\\ -215 & -120 & 725\end{bmatrix}$$

- $\det \underline{\underline{A}} = -562\,500 \neq 0$ a görbe nem elfajuló
- $\det \underline{\underline{A}}_{33} = \begin{vmatrix}59 & 12\\ 12 & 66\end{vmatrix} = 3750 > 0$ a görbe elliptikus

ELLIPSZIS

$$\left.\begin{aligned}59x_0+12y_0-215&=0\\ 12x_0+66y_0-120&=0\end{aligned}\right\}$$

A centrum koordinátái $x_0=\frac{17}{5}$; $y_0=\frac{6}{5}$

Laplace-egyenlet : $\det(\underline{\underline{A}}_{33}-\lambda\underline{\underline{1}})=0$

$$\begin{vmatrix}59-\lambda & 12\\ 12 & 66-\lambda\end{vmatrix}=0 ;$$

$\lambda^2-125\lambda+3750=0$

$\lambda_1=50$; $\lambda_2=75$

sajátértékek.

$\lambda_1=50$; $\begin{bmatrix}9 & 12\\ 12 & 16\end{bmatrix}\begin{bmatrix}r\\ s\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$; $3r+4s=0$; $\underline{t}_1=\frac{1}{5}\begin{bmatrix}4\\ -3\end{bmatrix}\alpha$

$\lambda_2=75$; $\begin{bmatrix}-16 & 12\\ 12 & -9\end{bmatrix}\begin{bmatrix}r\\ s\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$; $-4r+3s=0$; $\underline{t}_2=\frac{1}{5}\begin{bmatrix}3\\ 4\end{bmatrix}\alpha$

sajátvektorok

$M=(\underline{\underline{M}},\underline{m})$ a koordináta-rendszer elmozgatása

$\underline{\underline{M}}=\begin{bmatrix}\underline{t}_1 & \underline{t}_2\end{bmatrix}=\frac{1}{5}\begin{bmatrix}4 & 3\\ -3 & 4\end{bmatrix}$; $\underline{m}=\begin{bmatrix}x_0\\ y_0\end{bmatrix}=\frac{1}{5}\begin{bmatrix}17\\ 6\end{bmatrix}$ (lehetőleg $\det\underline{\underline{M}}=+1$

$B=(\underline{\underline{B}},\underline{b})$ koordináta transzformáció

$\underline{\underline{B}}=\underline{\underline{M}}^{-1}=\underline{\underline{M}}^{T}=\frac{1}{5}\begin{bmatrix}4 & -3\\ 3 & 4\end{bmatrix}$; $\underline{b}=-\underline{\underline{M}}^{-1}\underline{m}=\begin{bmatrix}2\\ 3\end{bmatrix}$

$\begin{bmatrix}u\\ v\end{bmatrix}=\frac{1}{5}\begin{bmatrix}4 & -3\\ 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}-\begin{bmatrix}2\\ 3\end{bmatrix}$; $\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}4 & 3\\ -3 & 4\end{bmatrix}\begin{bmatrix}u\\ v\end{bmatrix}+\frac{1}{5}\begin{bmatrix}17\\ 6\end{bmatrix}$

$x=\frac{1}{5}(4u+3v)+\frac{17}{5}$; $y=\frac{1}{5}(-3u+4v)+\frac{6}{5}$

behelyettesítve : $50u^2+75v^2-150=0$

$$\left(\frac{u}{\sqrt{3}}\right)^2+\left(\frac{v}{\sqrt{2}}\right)^2=1 \qquad \left.\begin{aligned}a&=\sqrt{3}\\ b&=\sqrt{2}\end{aligned}\right\} \Rightarrow c=1$$

fókuszok (u,v)-ben $\begin{bmatrix}\pm1\\ 0\end{bmatrix}$; (x,y)-ban pedig

$F_1\left(\frac{21}{5} ; \frac{3}{5}\right)$ $\qquad$ $F_2\left(\frac{13}{5} ; \frac{9}{5}\right)$

$$6xy + 8y^2 - 12x - 26y + 11 = 0$$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 3 & -6 \\ 3 & 8 & -13 \\ -6 & -13 & 11 \end{bmatrix}$$

$\det \underline{\underline{A}} = 81 \neq 0$

nem elfajuló a görbe

$\det \underline{\underline{A}}_{33} = \begin{vmatrix} 0 & 3 \\ 3 & 8 \end{vmatrix} = -9 < 0$

$\Longrightarrow$ HIPERBOLA

Centrum

$$\left.\begin{aligned} 0x_0 + 3y_0 - 6 &= 0 \\ 3x_0 + 8y_0 - 13 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_0 &= -1 \\ y_0 &= 2 \end{aligned}$$

$\underline{\underline{A}}_{33}$ sajátirányai: Laplace-egyenlet $\begin{vmatrix} -\lambda & 3 \\ 3 & 8-\lambda \end{vmatrix} = 0$

$\lambda^2 - 8\lambda - 9 = 0$; $\lambda_1 = 9$, $\lambda_2 = -1$

$\lambda_1 = 9 \quad \begin{bmatrix} -9 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $q = 3p$; $\underline{s}_1 = \frac{\alpha}{\sqrt{10}} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$;

$\lambda_2 = -1 \quad \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $p = -3q$; $\underline{s}_2 = \frac{\alpha}{\sqrt{10}} \begin{bmatrix} -3 \\ 1 \end{bmatrix}$;

$$A \sim \left(\begin{bmatrix} | & | \\ \underline{s}_1 & \underline{s}_2 \\ | & | \end{bmatrix}; \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \right) = \left(\overbrace{\frac{1}{\sqrt{10}} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}}^{\underline{\underline{A}}}, \overbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}^{\underline{a}} \right)$$

$$B \sim \left(\begin{bmatrix} - & \underline{s}_1^T & - \\ - & \underline{s}_2^T & - \end{bmatrix}, -\begin{bmatrix} - & \underline{s}_1^T & - \\ - & \underline{s}_2^T & - \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \right) = \left(\overbrace{\frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}}^{\underline{\underline{B}}}, \overbrace{-\frac{5}{\sqrt{10}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}^{\underline{b}} \right)$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \underline{\underline{B}} \begin{bmatrix} x \\ y \end{bmatrix} + \underline{b} \quad ; \quad \begin{bmatrix} x \\ y \end{bmatrix} = \underline{\underline{A}} \begin{bmatrix} u \\ v \end{bmatrix} + \underline{a}$$

Kanonikus egyenlet az utóbbiból

$$\left.\begin{aligned} x &= \frac{1}{\sqrt{10}} u - \frac{3}{\sqrt{10}} v - 1 \\ y &= \frac{3}{\sqrt{10}} u + \frac{1}{\sqrt{10}} v + 2 \end{aligned}\right\} \quad \begin{aligned} & 90u^2 - 10v^2 = 90 \\ & \frac{u^2}{1^2} - \frac{v^2}{3^2} = 1 \end{aligned}$$

$a = 1$; $b = 3$, $c = \sqrt{10}$

A fókuszok koordinátái $[u, v]$-ben $F_{12}(\pm\sqrt{10}, 0)$;

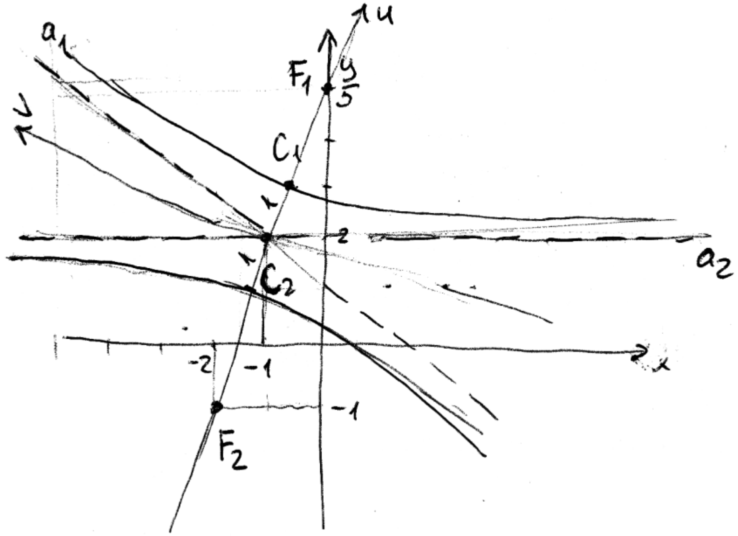
$[x, y]$-ban: $\underline{\underline{A}} \begin{bmatrix} \pm\sqrt{10} \\ 0 \end{bmatrix} + \underline{a} =$

$$= \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} \pm\sqrt{10} \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} =$$

$$= \frac{1}{\sqrt{10}} \begin{bmatrix} \pm\sqrt{10} \\ \pm 3\sqrt{10} \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \rightarrow \begin{aligned} & F_1(0, 5) \\ & F_2(-2, -1) \end{aligned}$$

Aszimptoták egyenlete $[u, v]$-ben: $v = \pm 3u$

$$\left.\begin{aligned} u &= \frac{1}{\sqrt{10}} x + \frac{3}{\sqrt{10}} y - \frac{5}{\sqrt{10}} \\ v &= \frac{-3}{\sqrt{10}} x + \frac{1}{\sqrt{10}} y - \frac{5}{\sqrt{10}} \end{aligned}\right\} \quad \begin{aligned} a_1 &: y = \frac{-3}{4} x + \frac{5}{4} \\ a_2 &: y = 2 \end{aligned}$$



$$4x^2+4xy+y^2+6\sqrt{5}x-2\sqrt{5}y+25=0$$

$$\underline{\underline{A}}=\begin{bmatrix}4 & 2 & 3\sqrt{5}\\ 2 & 1 & -\sqrt{5}\\ 3\sqrt{5} & -\sqrt{5} & 25\end{bmatrix} \qquad \det A\neq 0 \quad \det A_{33}=0 \quad \text{parabola}$$

laplace egy.

$$\begin{vmatrix}4-\lambda & 2\\ 2 & 1-\lambda\end{vmatrix}=0 \qquad \lambda^2-5\lambda=0 \qquad \begin{matrix}\lambda_1=0\\ \lambda_2=5\end{matrix}$$

$$\lambda=0 \quad \begin{bmatrix}4 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \qquad \begin{matrix}4x+2y=0\\ 2x+y=0\end{matrix} \qquad \begin{matrix}x=t\\ y=-2t\end{matrix} \qquad \frac{1}{\sqrt{5}}\begin{bmatrix}1\\ -2\end{bmatrix}$$

$$\lambda=5 \quad \begin{bmatrix}-1 & 2\\ 2 & -4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \qquad \begin{matrix}-x+2y=0\\ 2x-4y=0\end{matrix} \qquad \begin{matrix}x=2u\\ y=u\end{matrix} \qquad \frac{1}{\sqrt{5}}\begin{bmatrix}2\\ 1\end{bmatrix}$$

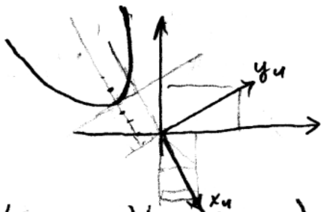
Báziscsere

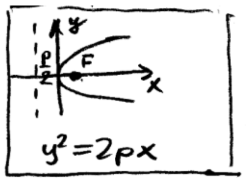
$$\underline{i}_u=\frac{1}{\sqrt{5}}\begin{bmatrix}1\\ -2\end{bmatrix} \qquad \underline{j}_u=\frac{1}{\sqrt{5}}\begin{bmatrix}2\\ 1\end{bmatrix} \qquad \underline{\underline{B}}=\frac{1}{\sqrt{5}}\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$$

$$\begin{bmatrix}x_u\\ y_u\end{bmatrix}=\frac{1}{\sqrt{5}}\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \qquad \begin{bmatrix}x\\ y\end{bmatrix}=\underline{\underline{B}}^{-1}\begin{bmatrix}x_u\\ y_u\end{bmatrix}=\frac{1}{\sqrt{5}}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}x_u\\ y_u\end{bmatrix}$$

$$x=\frac{1}{\sqrt{5}}(x_u+2y_u)$$

$$y=\frac{1}{\sqrt{5}}(-2x_u+y_u)$$





$$4\cdot\frac{1}{5}(x_u+2y_u)^2+4\cdot\frac{1}{5}(x_u+2y_u)(-2x_u+y_u)+\frac{1}{5}(-2x_u+y_u)^2$$

$$+6\sqrt{5}\cdot\frac{1}{\sqrt{5}}(x_u+2y_u)-2\sqrt{5}\frac{1}{\sqrt{5}}(-2x_u+y_u)+25=0$$

$$\frac{4}{5}(x_u^2+4x_uy_u+4y_u^2)+\frac{4}{5}(-2x_u^2-3x_uy_u+2y_u^2)+\frac{1}{5}(4x_u^2-4x_uy_u+y_u^2)$$

$$+10x_u+10y_u+25=$$

$$=0\cdot x_u^2+0x_uy_u+5y_u^2+10y_u+10x_u+25$$

$$5(y_u^2+2y_u+1)+2(x_u+2)=0$$

$$(y_u+1)^2=-2(x_u+2) \qquad p=1$$

csúcspont: $\qquad C_u(-2,-1)$

$$\underline{\underline{B}}^{-1}=\frac{1}{\sqrt{5}}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix} \qquad C\left(-\frac{4}{\sqrt{5}};\frac{3}{\sqrt{5}}\right)$$

fókusz

$$F_u\left(-\frac{5}{2},-1\right) \qquad F\left(-\frac{9}{2\sqrt{5}},\frac{4}{\sqrt{5}}\right)$$

direktrix

$$x_u=-\frac{3}{2}\ ; \quad y_u=t$$

$$x=-\frac{3}{2\sqrt{5}}+\frac{2}{\sqrt{5}}t \ ; \quad y=\frac{3}{\sqrt{5}}+\frac{1}{\sqrt{5}}t$$

$$\frac{\sqrt{5}}{2}\left(x+\frac{3}{2\sqrt{5}}\right)-\sqrt{5}\left(y-\frac{3}{\sqrt{5}}\right)=0 \ ; \quad x-2y=\frac{9}{2\sqrt{5}}$$