On a rough equivalence of the strong cosmic censor conjecture and the physical Church–Turing thesis

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Some variants of the cosmic censorship

The deepest open question of classical general relativity theory is the validity of the "cosmic censorship". From the mathematical viewpoint given an initial data set (S, h, k) we can ask for a space-time (M, g) as its

(i) *short time* evolution (or existence and uniqueness): very well understood;

(ii) *long time* evolution (or cosmic censorship): still "very much open" (Penrose, 1999).

Compared to (i) is (ii) merely technically more difficult or is (ii) even conceptually deeper?

Instead of having a complete solution several versions have appeared during the course of the past four decades.

WCCC *In a generic (i.e., stable), physically relevant (i.e., obeying some energy condition), asymptotically flat space-time singularities are hidden behind event horizons of black holes.*

SCCC *A generic (i.e., stable), physically relevant (i.e., obeying some energy condition) space-time is globally hyperbolic.*

Genericity or equivalently stability is very important: otherwise several counter-examples exist (e.g. Taub–NUT, anti-de Sitter, Reissner–Nordström, Kerr, etc.)! But it is very difficult to grasp the meaning of this concept.

Rather let us try to characterize the types of possible violations of the global hyperbolicity (Geroch–Horowitz, 1979; Penrose, 1979):

SCCC-GHP *If a physically relevant (i.e., obeying some energy condition) space-time is not globally hyperbolic then its Cauchy horizon looks like either that of the Taub–NUT or that of the Kerr space-time.*

Here the problematic term "genericity" or "stability" is avoided. However a mathematically precise formulation of this conjecture (cf. p.305 in R.M. Wald: General relativity, 1984) can be proved (Etesi, 2012) hence this version apparently cannot catch the full depth of SCCC.

The proof (not presented here) is motivated by the theory of Malament–Hogarth space-times and is based on seeking causal curves without future endpoint in the causal past of events along the future Cauchy horizon.

Extensions and curves

Definition

(M, g) is called a Malament–Hogarth space-time if there is a future-directed timelike half-curve $\gamma_C : \mathbb{R}^+ \to M$ such that $\|\gamma_C\| = +\infty$ and a point $q \in M$ satisfying $\gamma_C(\mathbb{R}^+) \subset J^-(q)$. The event $q \in M$ is called a Malament–Hogarth event.

Another timelike curve $\gamma_O : [0, b] \to M$ also exists such that $\|\gamma_O\| < +\infty$, $\gamma_O([0, b]) \subset J^-(q)$, $\gamma_O(0) = \gamma_C(0)$ and $\gamma_O(b) = q$. Then $(M, g, q, \gamma_C, \gamma_O)$ can be used for non-Turing computations (e.g. Hogarth, 1992, 1994; Etesi–Németi, 2002; Welch, 2008; etc.).

Theorem

If (M, g) is a Malament–Hogarth space-time then (M, g) is not globally hyperbolic. $\diamond$

Hence these space-times are subjects to SCCC.

Conversely: Which non-globally hyperbolic space-times are Malament–Hogarth ones?

(i) Malament–Hogarth space-times: any maximal extension of anti-de Sitter, Reissner–Nordström, Kerr, etc. (corresponding initial data set is asymptotically anti-de Sitter or flat). In this case of course there exist causal curves γ_C such that $\|\gamma_C\| = +\infty$ and an event q satisfying $\gamma_C(\mathbb{R}^+) \subset J^-(q)$;

(ii) Non-Malament–Hogarth ones: any maximal extension of the Taub–NUT, certain Gowdy space-times with toroidal spatial topology, etc. (corresponding initial data set is not like above). In this case still there exist causal curves γ_C without future endpoint such that $\|\gamma_C\| < +\infty$ and an event q satisfying $\gamma_C(\mathbb{R}^+) \subset J^-(q)$.

Consequently we are tempted (Etesi, 2002) to replace SCCC-GHP by

SCCC-MH *If a physically relevant (i.e., obeying some energy condition), asymptotically flat or asymptotically hyperbolic (i.e., anti-de Sitter) space-time is not globally hyperbolic then it is a Malament–Hogarth space-time.*

Note that this version—like SCCC-GHP—avoids the question of genericity or stability.

This SCCC-MH is open. A promissing way to attack it is to study the so-called radiation problem of general relativity (Christodoulou–O'Murchadha, 1981).

The problem of genericity or stability

Case-by-case studies demonstrate that all physically relevant solutions (M, g) of the Einstein constraint equations which are non-globally hyperbolic—i.e., arise as further extensions of maximal Cauchy developments of initial data sets (S, h, k)—are unstable: an arbitrary "small" but "generic" perturbation of their metric in the maximal Cauchy development destroys its extendibility across the Cauchy horizon. That is, a generic perturbation of these non-globally hyperbolic space-times makes them globally hyperbolic.

This is in particular true for certain physically relevant Malament–Hogarth space-times: small perturbation of them (e.g. taking into account the effect of the computer itself on the geometry, etc.) destroys the Malament–Hogarth property.

That is, if SCCC is true then all physically relevant artificial computing systems based on Malament–Hogarth space-times i.e., capable for non-Turing computations contain an inherent instability.

Other artificial computing systems in principle capable for non-Turing computations (e.g. the generalized quantum computers of Calude–Pavlov, 2002; Kieu, 2003, 2004; etc.) also seem to suffer from inherent instabilities.

Even worse: artificial computing systems used just for very complex but still Turing computations (e.g. usual quantum computers or the Chern–Simons TQFT computer of Freedman, 1998 to calculate the Jones polynomial of knots) apparently contain an inherent instability increasing with the degree of computational complexity.

A proposed rough equivalence of two deep conjectures

Definition

A quintuple $(M, g, q, \gamma_C, \gamma_O)$ is called a gravitational computer if (M, g) is a space-time, γ_C, γ_O are timelike curves and $q \in M$ is an event such that the images of the curves lie within $J^-(q)$.

This concept is broad enough to serve as an abstract model for all kind of artificial computing systems based on classical physics so that an artificial computing system can perform non-Turing computations if and only if the corresponding gravitational computer is defined in an ambient space-time (M, g) possessing the Malament–Hogarth property.

Consider the following democratic version of the physical Church–Turing thesis:

Ph-ChT *An artificial computing system based on a generic (i.e., stable), relevant (i.e., obeying some energy condition) classical physical system realizes Turing-computable functions.*

Moreover recall

SCCC *A generic (i.e., stable), physically relevant (i.e., obeying some energy condition) space-time is globally hyperbolic.*

Accepting (i) *SCCC-MH* and (ii) *all artificial computing systems based on classical physics can be modeled by a gravitational computer* as above, we can see that SCCC and Ph-ChT are roughly equivalent. This could explain the permanent difficulty present in all approaches to both SCCC and Ph-ChT.

For further details please check:

arXiv: 1205.4550 [gr-qc]

or the more upgraded version

URL: http://www.math.bme.hu/~etesi/preprint.html

THANK YOU!