Matematika képletgyüjtemény
A2 Építőmérnök BSc képzés, 1. zh anyaga

$\sin^2 x + \cos^2 x = 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$
$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\cos^2 x = \frac{1 + \cos 2x}{2}$
$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$
$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$
$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$
$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
$\cosh x = \frac{e^x + e^{-x}}{2}$
$\sinh x = \frac{e^x - e^{-x}}{2}$
$\cosh^2 x - \sinh^2 x = 1$
$\sinh 2x = 2 \sinh x \cosh x$
$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\cosh^2 x = \frac{\cosh x + 1}{2}$
$\sinh^2 x = \frac{\cosh x - 1}{2}$

Differenciálási szabályok:

$(cf(x))' = cf'()x$, ahol c egy konstans
$(f(x) \pm g(x))' = f'(x) \pm g(x)'$
$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
$(f(g(x)))' = f'(g(x))g'(x)$

Integrálási szabályok:

$\int cf(x)dx = c \int f(x)dx$, ahol c egy konstans
$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$
$\int f(x)dx + c \quad \Rightarrow \int f(ax+b)dx = \frac{F(ax+b)}{a} + c$
$\int f^\alpha(x) f'(x)dx = \frac{f^{\alpha+1}}{\alpha+1} + c$, ha $\alpha \neq -1$
$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c$
$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$

$(x^n)' = nx^{n-1}$
$e^x = e^x$
$(a^x)' = a^x \ln a$
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \frac{1}{\cos^2 x}$
$(\cot x)' = -\frac{1}{\sin^2 x}$
$(\sinh x)' = \cosh x$
$(\cosh x)' = \sinh x$
$(\ln x)' = \frac{1}{x}$
$(\log_a x)' = \frac{1}{x \ln a}$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(arccotx)' = -\frac{1}{1+x^2}$
$(arsinhx)' = \frac{1}{\sqrt{1+x^2}}$
$(arcoshx)' = \frac{1}{\sqrt{x^2-1}}$
$(artanhx)' = \frac{1}{1-x^2}$
$(arcothx)' = \frac{1}{1-x^2}$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$
$\int \frac{1}{x}dx = \ln|x| + c$
$\int a^x dx = \frac{a^x}{\ln a} + c$
$\int \sin x dx = -\cos x + c$
$\int \cos x dx = \sin x + c$
$\int \tan x dx = \ln|\cos x| + c$
$\int \cot x dx = -\ln|\sin x| + c$
$\int \frac{1}{\cos^2 x}dx = \tan x + c$
$\int \frac{1}{\sin^2 x}dx = -\cot x + c$
$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\arctan\frac{x}{a} + c$
$\int \frac{1}{x^2-a^2}dx = \begin{cases} \frac{1}{a}artanh\frac{x}{a} + c, & ha \quad \left|\frac{x}{a}\right| < 1 \\ \frac{1}{a}arcoth\frac{x}{a} + c, & ha \quad \left|\frac{x}{a}\right| > 1 \end{cases}$
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + c$
$\int \frac{1}{\sqrt{a^2+x^2}}dx = arsinh\frac{x}{a} + c$
$\int \frac{1}{\sqrt{x^2-a^2}}dx = arcosh\frac{x}{a} + c$

Az $f(x)$ függvény $x = a$ helyen vett Taylor-sora:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Nevezetes függvények Maclaurin-sora ($a = 0$):
$e^x = 1 + x + \frac{x^2}{2!} + \dots \frac{x^n}{n!} + \dots,\ x \in \mathbb{R}$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - + \dots,\ x \in \mathbb{R}$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - + \dots,\ x \in \mathbb{R}$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - + \dots$, ha $-1 < x < 1$
$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - + \dots$, ha $-1 < x < 1$
$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \dots$

A 2π szerint periodikus $f(x)$ függvény Fourier-sora:

$$a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx),$$

ahol

$$a_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} f(x)dx,$$

$$a_k = \frac{1}{\pi}\int\limits_0^{2\pi} f(x)\cos kx dx,$$

$$b_k = \frac{1}{\pi}\int\limits_0^{2\pi} f(x)\sin kx dx.$$

Ha $f(x)$ egy páros függvény, akkor $b_k = 0$,

$$a_0 = \frac{1}{\pi}\int\limits_0^{\pi} f(x)dx, \quad a_k = \frac{2}{\pi}\int\limits_0^{\pi} f(x)\cos kx dx.$$

Ha $f(x)$ egy páratlan függvény, akkor $a_0 = a_k = 0$ és

$$b_k = \frac{2}{\pi}\int\limits_0^{\pi} f(x)\sin kx dx.$$

A T szerint periodikus $f(x)$ függvény Fourier-sora:

$$a_0 + \sum_{k=1}^{\infty}(a_k \cos\frac{2k\pi x}{T} + b_k \sin\frac{2k\pi x}{T}),$$

ahol

$$a_0 = \frac{1}{T}\int\limits_0^{T} f(x)dx,$$

$$a_k = \frac{2}{T}\int\limits_0^{T} f(x)\cos\frac{2k\pi x}{T}dx$$

$$b_k = \frac{2}{T}\int\limits_0^{T} f(x)\sin\frac{2k\pi x}{T}dx.$$