Formula sheet

Mathematics A2, English course, Midterm Test 1

$\sin^2 x + \cos^2 x = 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\mathrm{tg}(2x) = \frac{2\mathrm{tg}x}{1-\mathrm{tg}^2 x}$
$\sin^2 x = \frac{1-\cos 2x}{2}$
$\cos^2 x = \frac{1+\cos 2x}{2}$
$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$
$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$
$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$
$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$
$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
$\mathrm{ch}x = \frac{e^x + e^{-x}}{2}$
$\mathrm{sh}x = \frac{e^x - e^{-x}}{2}$
$\mathrm{ch}^2 x - \mathrm{sh}^2 x = 1$
$\mathrm{sh}2x = 2\mathrm{sh}x\mathrm{ch}x$
$\mathrm{ch}2x = \mathrm{ch}^2 x + \mathrm{sh}^2 x$
$\mathrm{ch}^2 x = \frac{\mathrm{ch}2x + 1}{2}$
$\mathrm{sh}^2 x = \frac{\mathrm{ch}2x - 1}{2}$

Rules of Differentiation:

$(cf(x))' = cf'(x)$, $c \in \mathbb{R}$
$(f(x) \pm g(x))' = f'(x) \pm g'(x)$
$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$
$(f(g(x)))' = f'(g(x))g'(x)$

Derivatives:

$(x^n)' = nx^{n-1}$
$(e^x)' = e^x$
$(a^x)' = a^x \ln a$
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\mathrm{tg}x)' = \frac{1}{\cos^2 x}$
$(\mathrm{ctg}x)' = -\frac{1}{\sin^2 x}$
$(\mathrm{sh}x)' = \mathrm{ch}x$
$(\mathrm{ch}x)' = \mathrm{sh}x$
$(\ln x)' = \frac{1}{x}$
$(\log_a x)' = \frac{1}{x \ln a}$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(\mathrm{arcctg}x)' = -\frac{1}{1+x^2}$
$(\mathrm{arsh}x)' = \frac{1}{\sqrt{1+x^2}}$
$(\mathrm{arch}x)' = \frac{1}{\sqrt{x^2-1}}$
$(\mathrm{artgh}x)' = \frac{1}{1-x^2}$
$(\mathrm{arctgh}x)' = \frac{1}{1-x^2}$

Rules of Integration:

$\int cf(x)dx = c \int f(x)dx$, minden $c \in \mathbb{R}$ esetén
$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$
$\int f(x)dx + c \quad \Rightarrow \int f(ax+b)dx = \frac{F(ax+b)}{a} + c$
$\int f^\alpha(x) f'(x)dx = \frac{f^{\alpha+1}}{\alpha+1} + c$, ha $\alpha \neq -1$
$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$
$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$

Indefinite Integrals:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$
$\int \frac{1}{x}dx = \ln|x| + c$
$\int a^x dx = \frac{a^x}{\ln a} + c$
$\int \sin x dx = -\cos x + c$
$\int \cos x dx = \sin x + c$
$\int \mathrm{tg}x dx = -\ln|\cos x| + c$
$\int \mathrm{ctg}x dx = \ln|\sin x| + c$
$\int \frac{1}{\cos^2 x}dx = \mathrm{tg}x + c$
$\int \frac{1}{\sin^2 x}dx = -\mathrm{ctg}x + c$
$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\mathrm{arctg}\frac{x}{a} + c$
$\int \frac{1}{x^2-a^2}dx = \begin{cases} \frac{1}{a}\mathrm{artgh}\frac{x}{a} + c, & ha \quad \left|\frac{x}{a}\right| < 1 \\ \frac{1}{a}\mathrm{arctgh}\frac{x}{a} + c, & ha \quad \left|\frac{x}{a}\right| > 1 \end{cases}$
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + c$
$\int \frac{1}{\sqrt{a^2+x^2}}dx = \mathrm{arsh}\frac{x}{a} + c$
$\int \frac{1}{\sqrt{x^2-a^2}}dx = \mathrm{arch}\frac{x}{a} + c$

Taylor series of the function $f(x)$ centered at $x = a$:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Famous McLaurin series ($a = 0$):

$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots,\ x \in \mathbb{R}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - + \dots,\ x \in \mathbb{R}$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - + \dots,\ x \in \mathbb{R}$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - + \dots,$ if $-1 < x < 1$

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - + \dots,$ if $-1 < x < 1$

$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \dots$

$$\text{if } -1 < x < 1$$

Fourier series of the periodic function $f(x)$ with period 2π:

$$a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx),$$

where

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx, \quad a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx,$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx.$$

If $f(x)$ is an even function then $b_k = 0$,

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)dx, \quad a_k = \frac{2}{\pi}\int_0^{\pi} f(x)\cos kx dx.$$

If $f(x)$ is an odd function then $a_0 = a_k = 0$ and

$$b_k = \frac{2}{\pi}\int_0^{\pi} f(x)\sin kx dx.$$

Fourier series of the periodic function $f(x)$ with period T:

$$a_0 + \sum_{k=1}^{\infty}(a_k \cos \frac{2k\pi x}{T} + b_k \sin \frac{2k\pi x}{T}),$$

where

$$a_0 = \frac{1}{T}\int_0^{T} f(x)dx, \quad a_k = \frac{2}{T}\int_0^{T} f(x)\cos \frac{2k\pi x}{T} dx$$

$$b_k = \frac{2}{T}\int_0^{T} f(x)\sin \frac{2k\pi x}{T} dx.$$